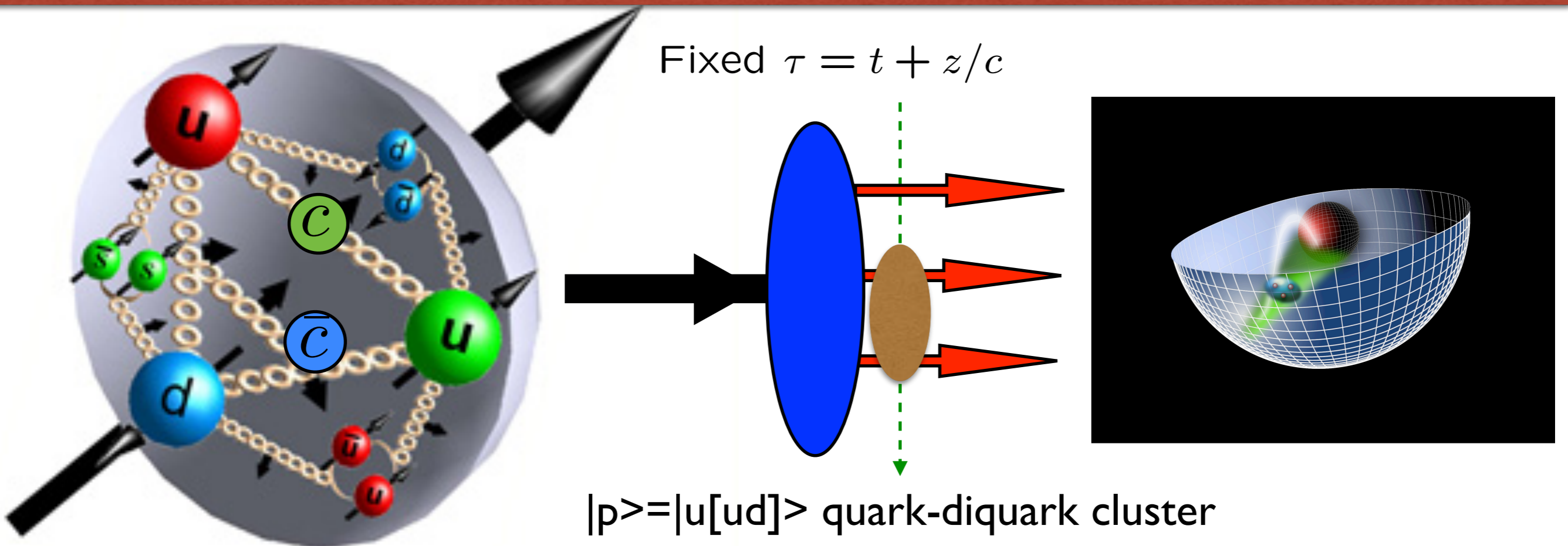


# Color Confinement and Supersymmetric Features of Hadron Physics from Light-Front Holography and Novel Features of QCD from Light-Front Holography II



with Guy de Tèramond, Hans Günter Dosch, Alexandre Deur, Marina Nielsen, Ivan Schmidt, F. Navarra, Jennifer Rittenhouse West, G. Miller, Keh-Fei Liu, Tianbo Llu, Liping Zou, S. Groote, Joshua Erlich, S. Koshkarev, Xing-Gang Wu, Sheng-Quan Wang, Cedric Lorcè, R. S. Sufian, R. Vogt, G. Lykasov, S. Gardner, S. Liuti, V. Lyubovitskij, L. di Giustino

**Bled Workshop**

What Comes  
Beyond the Standard  
Models?

Stan Brodsky

**SLAC**

NATIONAL  
ACCELERATOR  
LABORATORY



**Talk II July 8, 2021**

$M^2$  (GeV<sup>2</sup>)

bosons

fermions

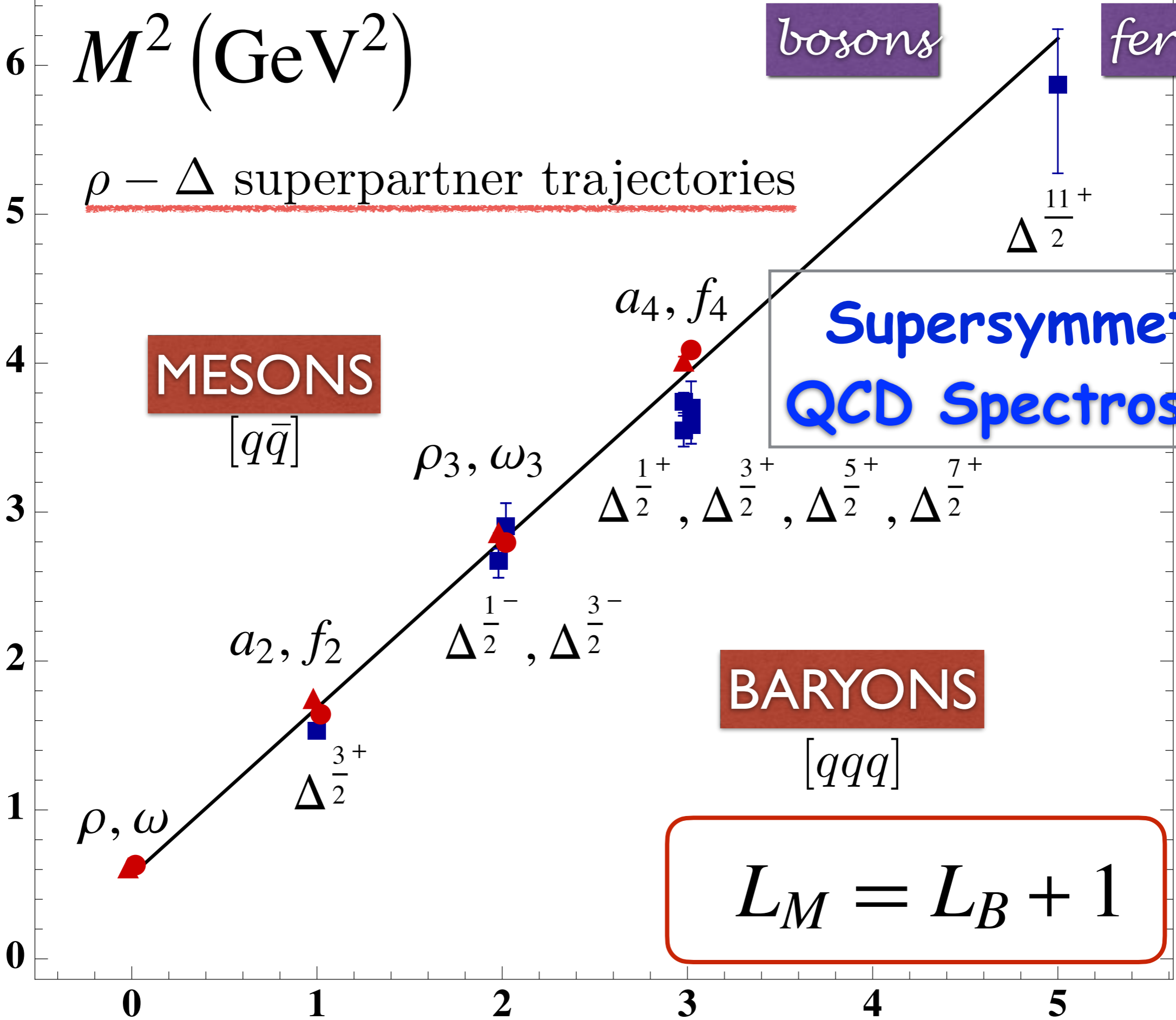
$\rho - \Delta$  superpartner trajectories

MESONS  
[ $q\bar{q}$ ]

Supersymmetric  
QCD Spectroscopy

BARYONS  
[ $qqq$ ]

$L_M = L_B + 1$

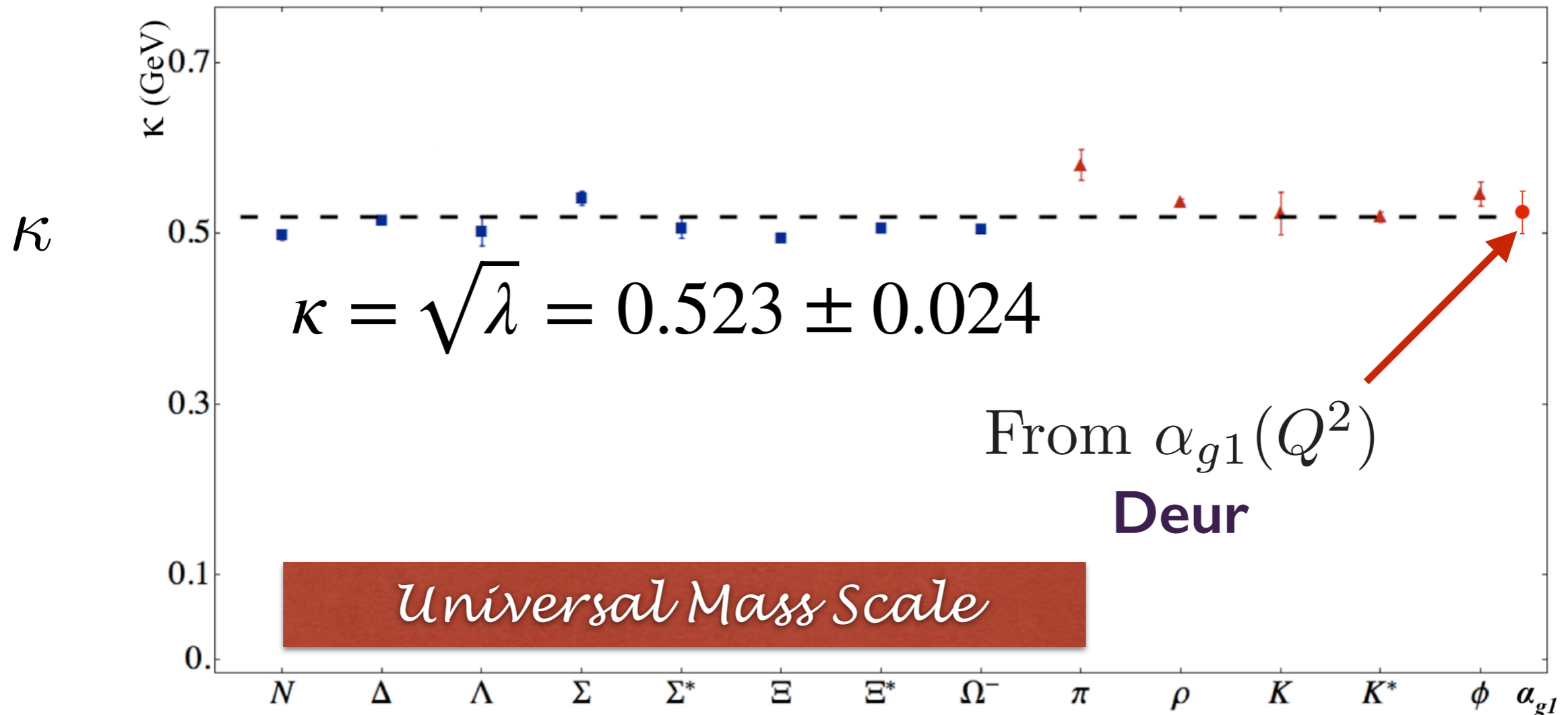




$$\lambda = \kappa^2$$

*de Tèramond, Dosch, Lorce', sjb*

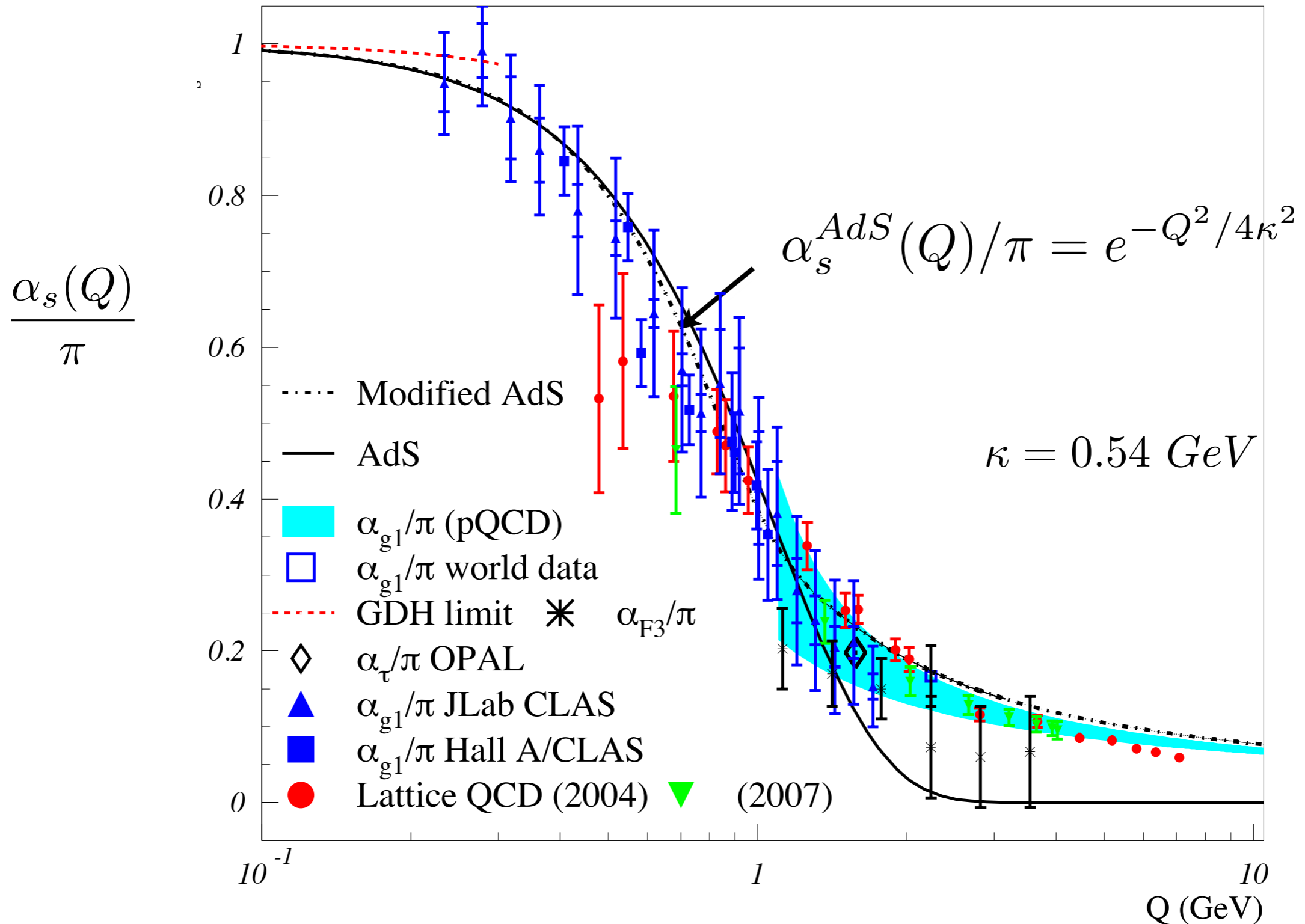
$$m_u = m_d = 46 \text{ MeV}, m_s = 357 \text{ MeV}$$



**Fit to the slope of Regge trajectories,  
including radial excitations**

**Same Regge Slope for Meson, Baryons:  
Supersymmetric feature of hadron physics**

# Analytic, defined at all scales, IR Fixed Point



**AdS/QCD dilaton predicts the nonperturbative corrections to the QCD running coupling**

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

**Deur, de Teramond, sjb**



# Challenge: Compute Hadron Structure, Spectroscopy, and Dynamics from QCD!

- **Color Confinement**
- **Origin of the QCD Mass Scale**
- **Meson and Baryon Spectroscopy**
- **Universal Regge Slopes:  $n$  and  $L$  for Mesons, Baryons, Tetraquarks**
- **Almost Massless Pion: GMOR Chiral Symmetry Breaking**  
$$M_\pi^2 f_\pi^2 = -\frac{1}{2}(m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle + \mathcal{O}((m_u + m_d)^2)$$
- **QCD Coupling at all Scales  $\alpha_s(Q^2)$**
- **Eliminate Scale Uncertainties and Scheme Dependence**
- **Heavy Quark Distributions**

$$\mathcal{L}_{QCD} \rightarrow \psi_n^H(x_i, \vec{k}_{\perp i}, \lambda_i) \quad \text{Valence and Higher Fock States}$$

# Supersymmetry in QCD

- A hidden symmetry of Color  $SU(3)_c$  in hadron physics
- QCD: No squarks or gluinos!
- Emerges from Light-Front Holography and Super-Conformal Algebra
- Color Confinement
- Massless Pion in Chiral Limit

*de Téramond, Dosch, Lorcé, sjb*

*Stan Brodsky  
Bled Workshop*

*Supersymmetric Features of Hadron Physics  
from Superconformal Algebra  
and Light-Front Holography*



6 July 2021



*Need a First Approximation to QCD*

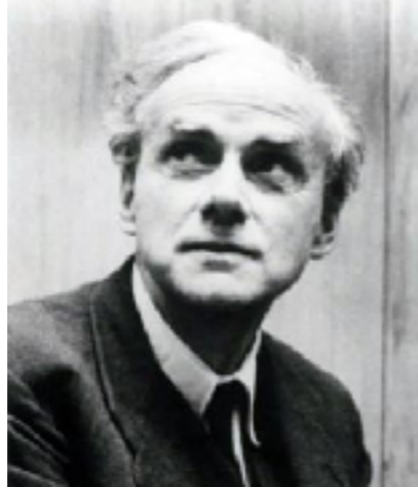
*Comparable in simplicity to  
Schrödinger Theory in Atomic Physics*

**Relativistic, Frame-Independent, Color-Confining**

**Origin of hadronic mass scale**

*AdS/QCD  
Light-Front Holography  
Superconformal Algebra*

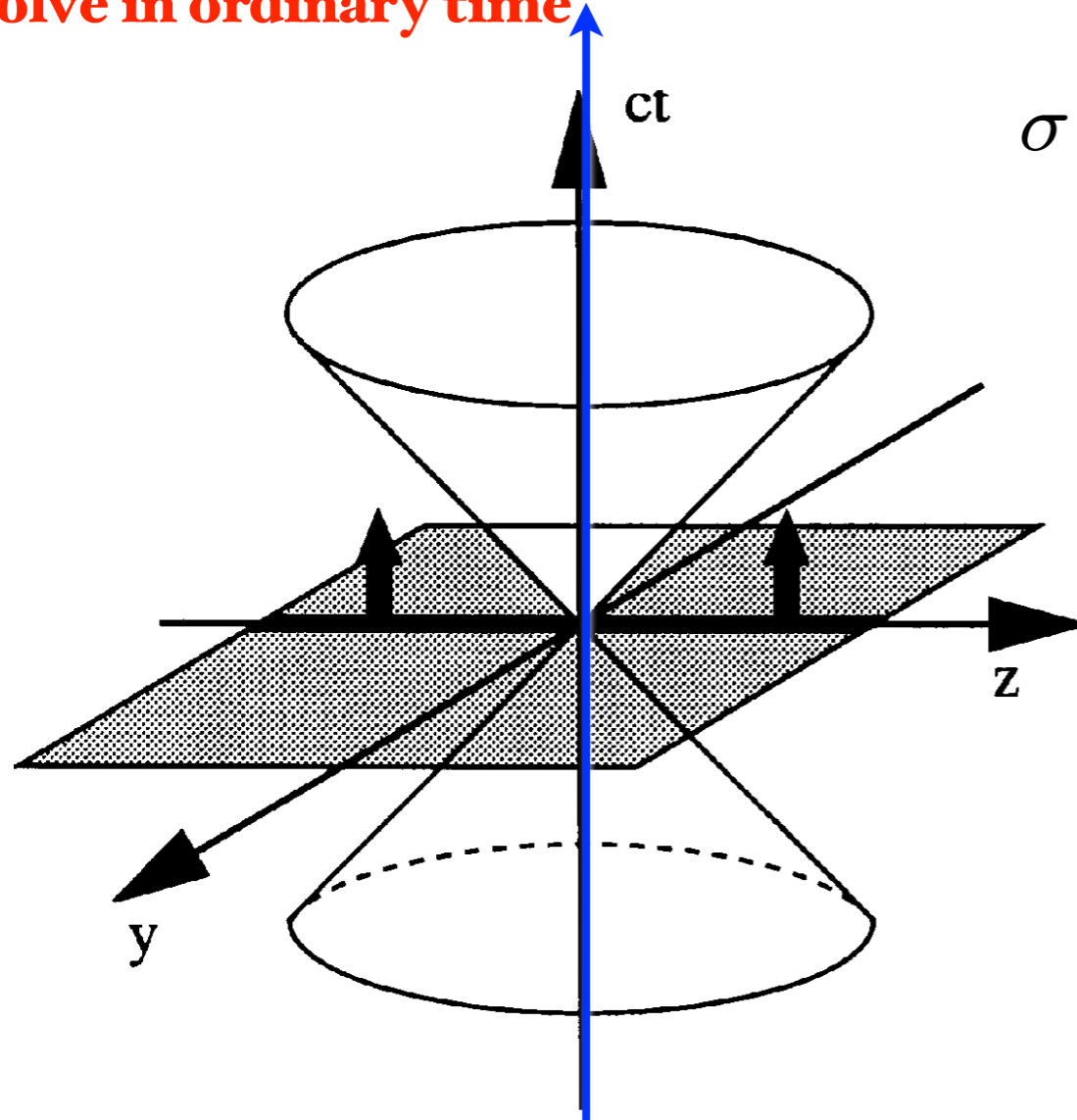
*No parameters except for quark masses!*



**P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)**

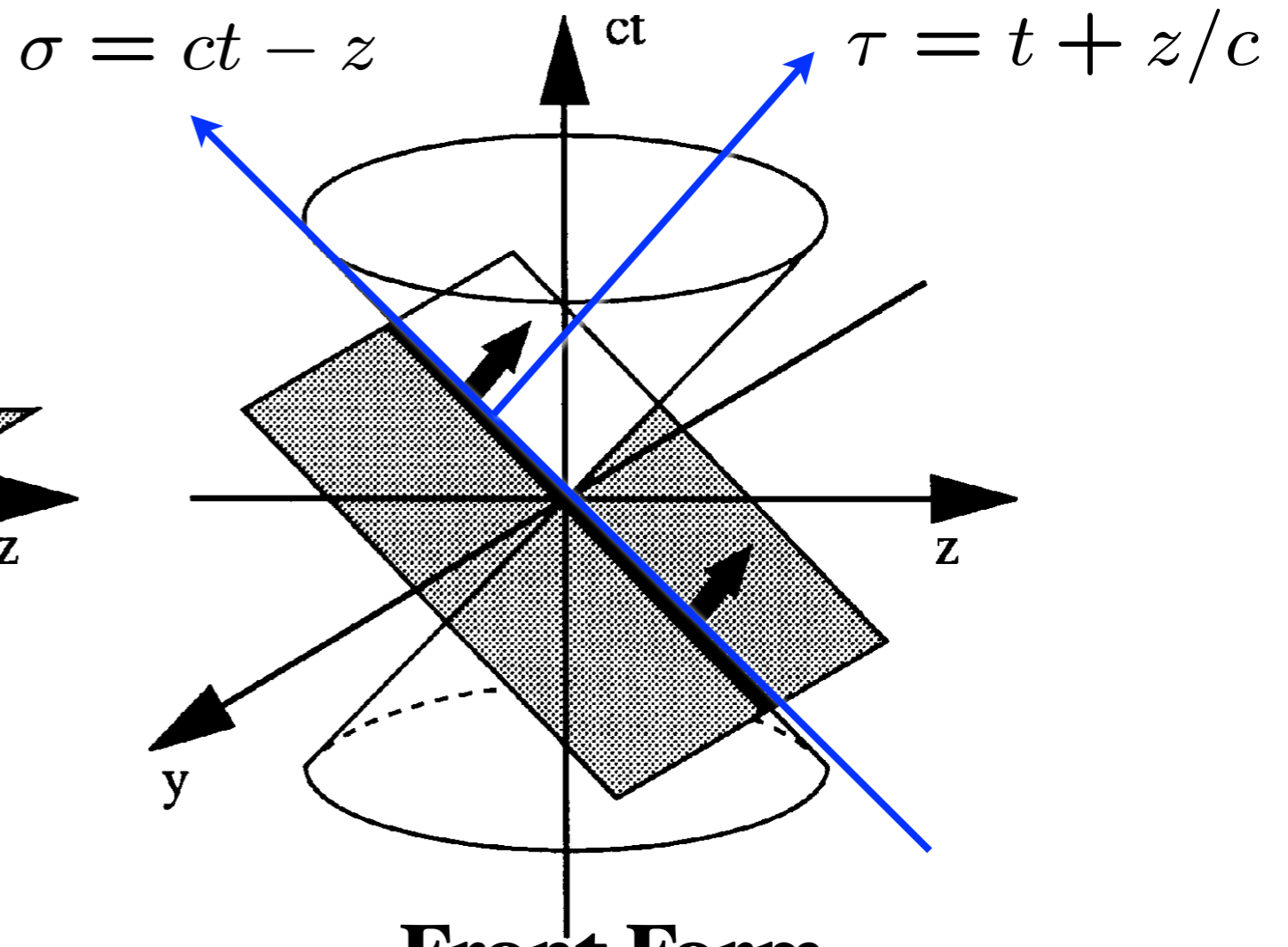
*Dirac's Amazing Idea:  
The "Front Form"*

**Evolve in ordinary time**



**Instant Form**

**Evolve in light-front time!**



**Front Form**

*Casual, Boost Invariant!*

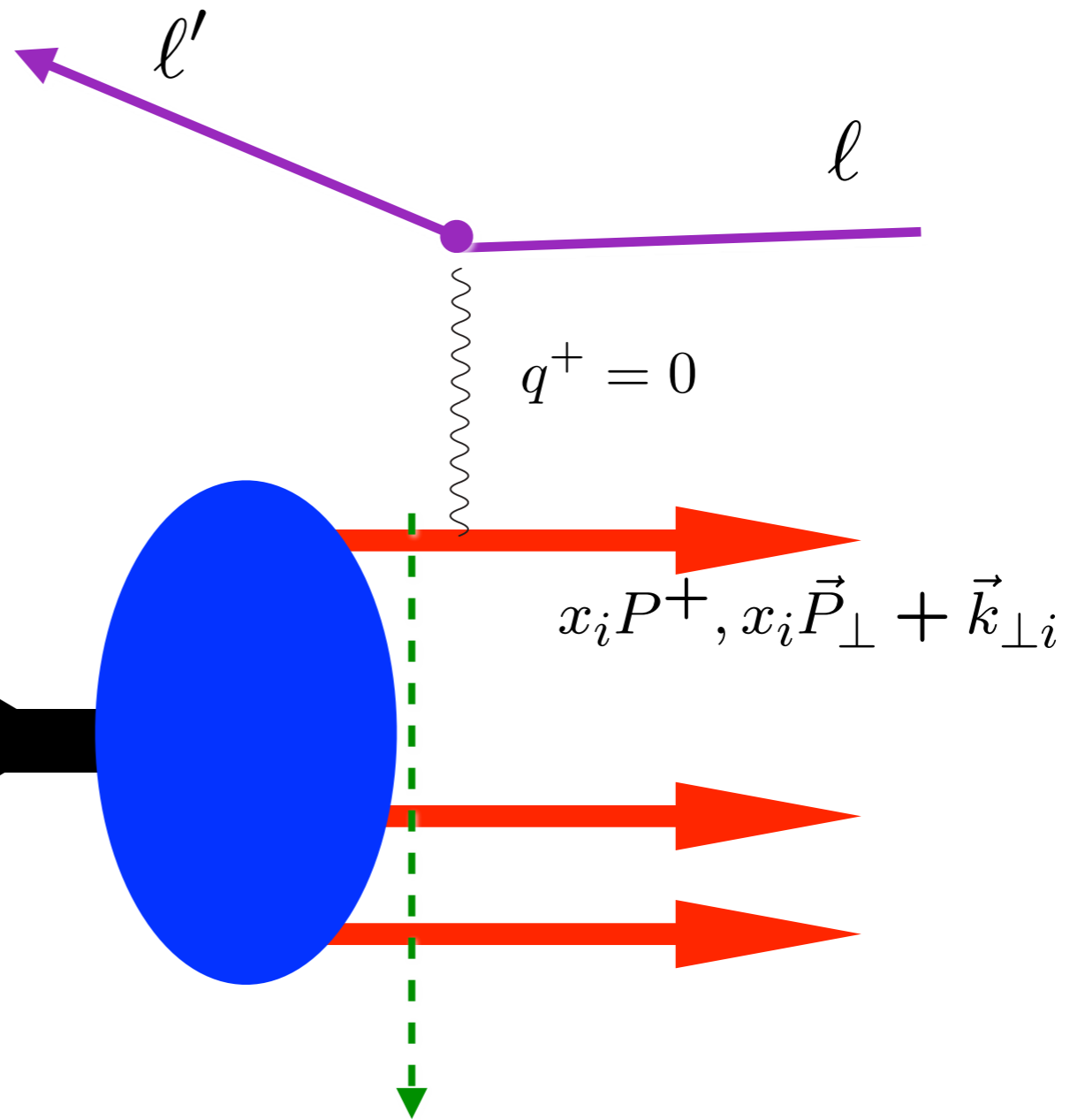
● **Trivial LF Vacuum (up to zero modes)**

Comparing light-front quantization with instant-time quantization  
 Philip D. Mannheim(Connecticut U.),  
 Peter Lowdon(Ecole Polytechnique, CPHT),  
 Stanley J. Brodsky(SLAC)

• e-Print: [2005.00109](https://arxiv.org/abs/2005.00109) [hep-ph]



$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



## Dirac: Front Form

*Measurements of hadron LF wavefunction are at fixed LF time*

Fixed  $\tau = t + z/c$

*Like a flash photograph*

$$x_{bj} = x = \frac{k^+}{P^+}$$

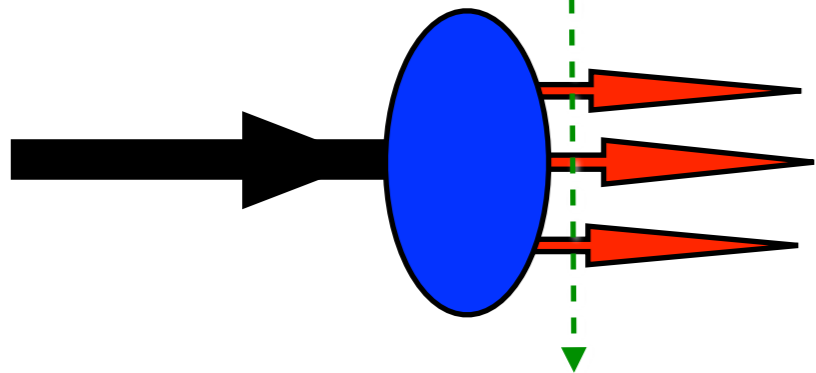
*Invariant under boosts! Independent of  $P^\mu$*

# Bound States in Relativistic Quantum Field Theory:

## *Light-Front Wavefunctions*

Dirac's Front Form: Fixed  $\tau = t + z/c$

Fixed  $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

***Invariant under boosts. Independent of  $P^\mu$***

$$H_{LF}^{QCD} |\psi\rangle = M^2 |\psi\rangle$$

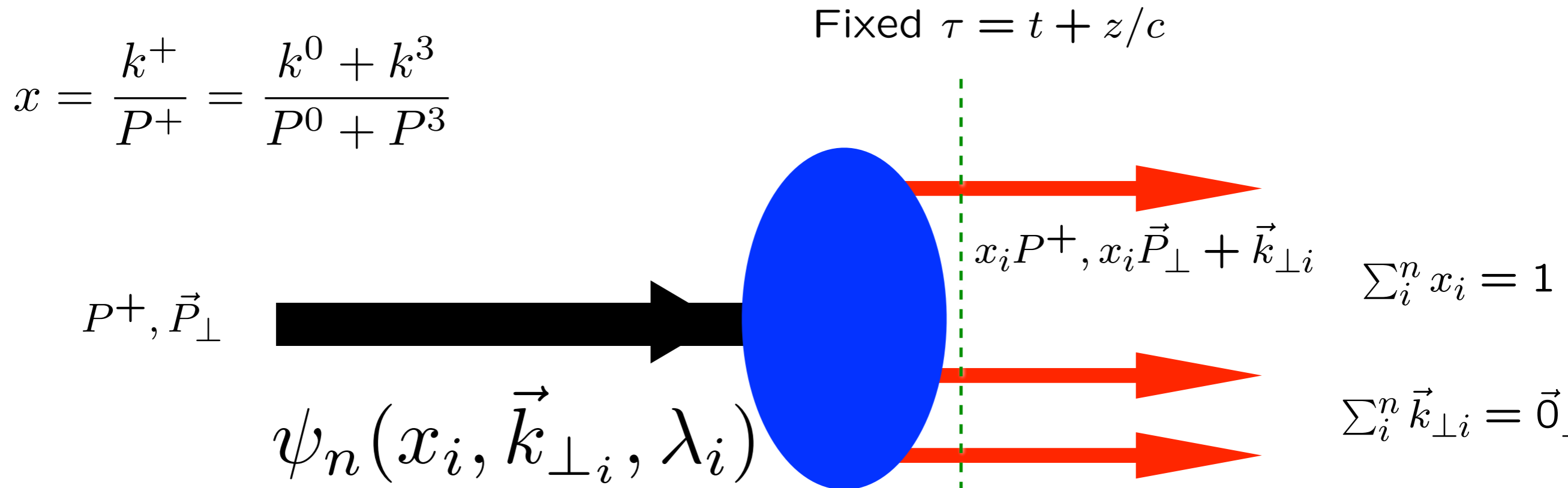
**Direct connection to QCD Lagrangian**

***LF Wavefunction: off-shell in invariant mass***

*Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space*



# Light-Front Wavefunctions: **rigorous** representation of composite systems in quantum field theory



*Eigenstate of LF Hamiltonian*

$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

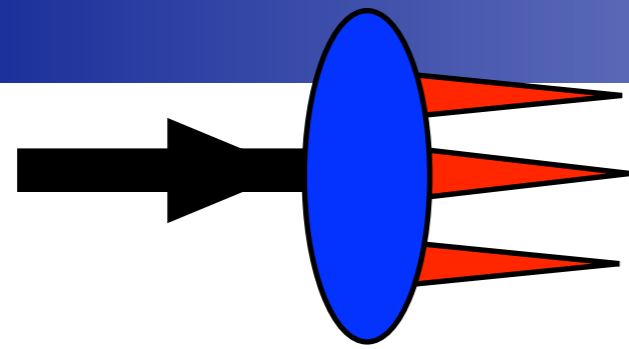
$$|p, J_z\rangle = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i\rangle$$

*Invariant under boosts! Independent of  $P^\mu$*

**Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS**

*Lorce,  
Pasquini*

**Light-Front Wavefunctions  
underly hadronic observables**



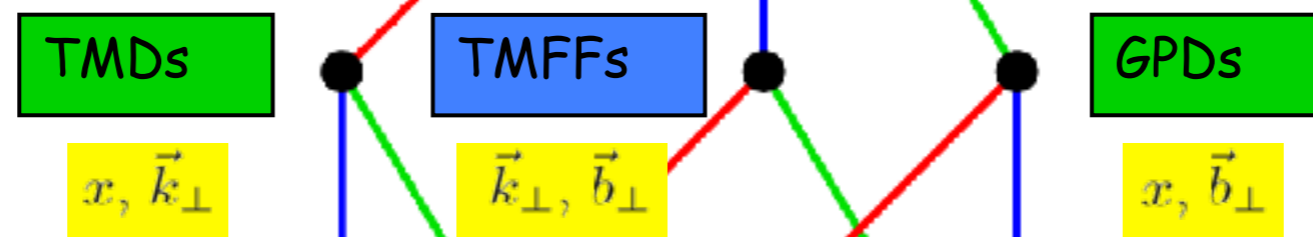
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Transverse density in  
momentum space

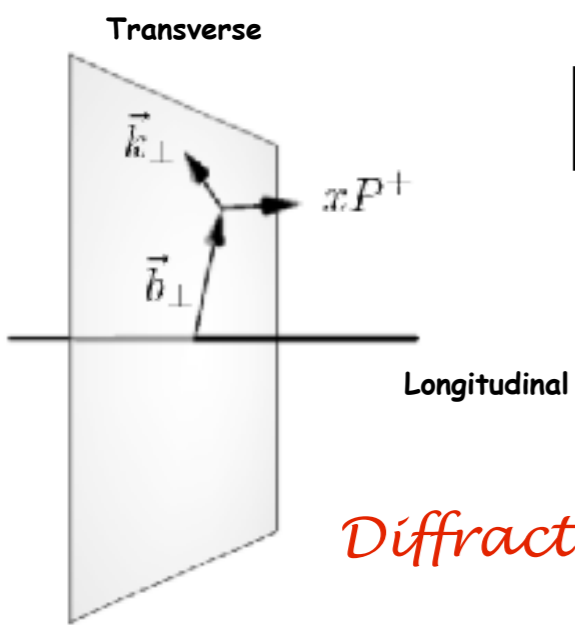
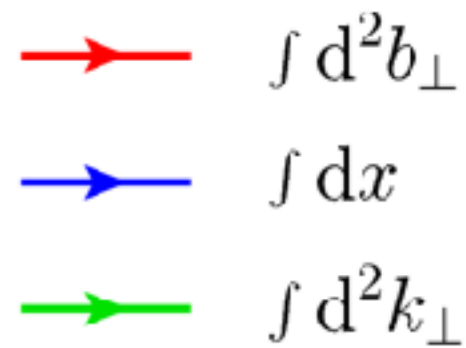
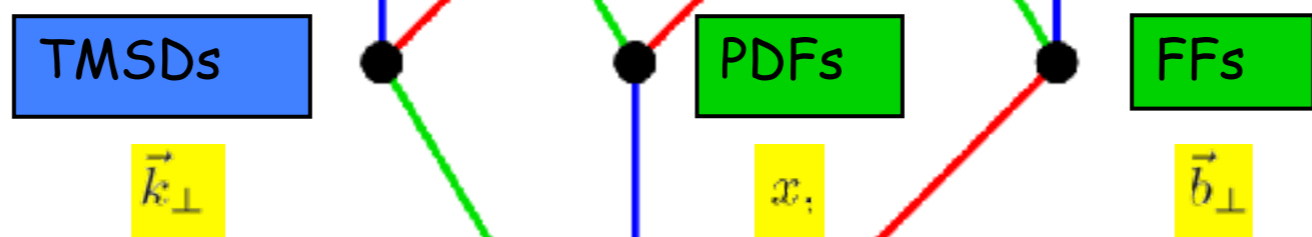
Momentum space  $\vec{k}_{\perp} \leftrightarrow \vec{z}_{\perp}$  Position space  
 $\vec{\Delta}_{\perp} \leftrightarrow \vec{b}_{\perp}$

Transverse density in position  
space

**Weak transition  
form factors**



**DGLAP, ERBL Evolution  
Factorization Theorems**



*Diffractive DIS from FSI*

*Sivers, T-odd from lensing*

**Charges**

## *Two Definitions of Vacuum State*

**Instant Form: Lowest Energy Eigenstate of Instant-Form Hamiltonian**

$$H|\psi_0\rangle = E_0|\psi_0\rangle, E_0 = \min\{E_i\}$$

*Eigenstate defined at one time  $t$  over all space;  
Acausal! Frame-Dependent*

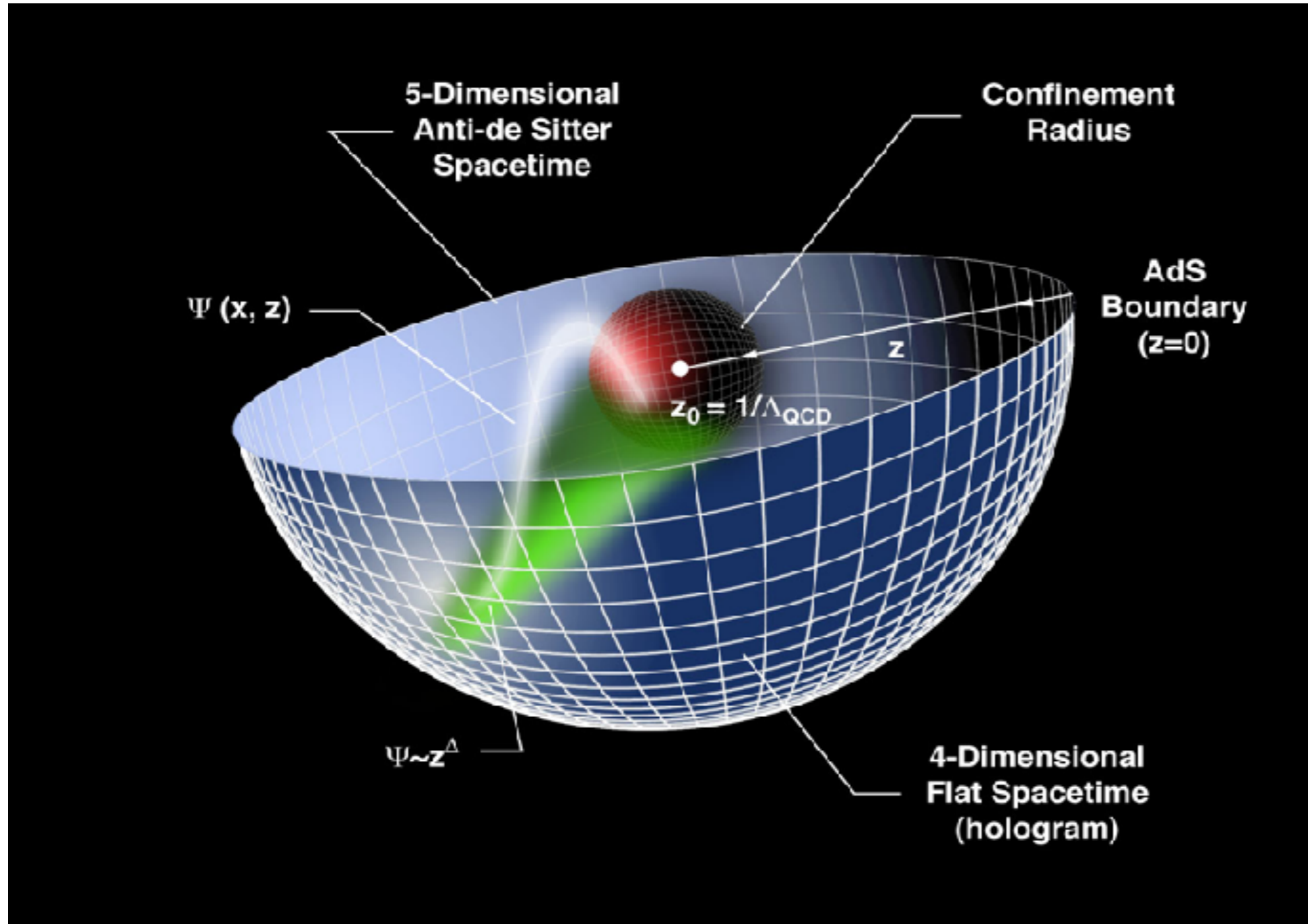
**Front Form: Lowest Invariant Mass Eigenstate of Light-Front Hamiltonian**

$$H_{LF}|\psi_0\rangle_{LF} = M_0^2|\psi_0\rangle_{LF}, M_0^2 = 0.$$

*Frame-independent eigenstate at fixed LF time  $\tau = t+z/c$   
within causal horizon*

*Frame-independent description of the causal physical universe!*

# AdS<sub>5</sub>



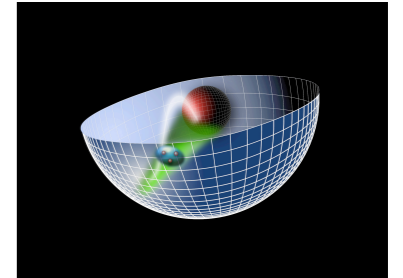
*Changes in physical length scale mapped to evolution in the 5th dimension z*

**in collaboration with Guy de Teramond and H. Guenter Dosch**



# Dilaton-Modified AdS

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2} (\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$



- **Soft-wall dilaton profile breaks conformal invariance**  $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement in z**
- **Introduces confinement scale  $\kappa$**
- **Uses AdS<sub>5</sub> as template for conformal theory**

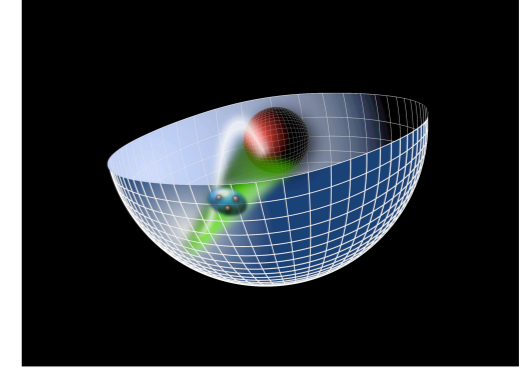
*Stan Brodsky*  
*Bled Workshop*

Supersymmetric Features of Hadron Physics  
from Superconformal Algebra  
and Light-Front Holography



7 July 2021

# AdS<sub>5</sub>



- Isomorphism of  $SO(4, 2)$  of **conformal QCD** with the group of **isometries** of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

*invariant measure* ←

$x^\mu \rightarrow \lambda x^\mu, z \rightarrow \lambda z$ , maps scale transformations into the holographic coordinate  $z$ .

- AdS mode in  $z$  is the extension of the hadron wf into the fifth dimension.
- Different values of  $z$  correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$ : invariant separation between quarks

- The AdS boundary at  $z \rightarrow 0$  correspond to the  $Q \rightarrow \infty$ , UV zero separation limit.

## AdS/CFT

## Holographic Mapping of AdS Modes to QCD LFWFs

*Drell-Yan-West: Form Factors are  
Convolution of LFWFs*

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0 \left( \zeta q \sqrt{\frac{1-x}{x}} \right) \tilde{\rho}(x, \zeta),$$

with  $\tilde{\rho}(x, \zeta)$  QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$

- Compare AdS and QCD expressions of FFs for arbitrary  $Q$  using identity:

$$\int_0^1 dx J_0 \left( \zeta Q \sqrt{\frac{1-x}{x}} \right) = \zeta Q K_1(\zeta Q),$$

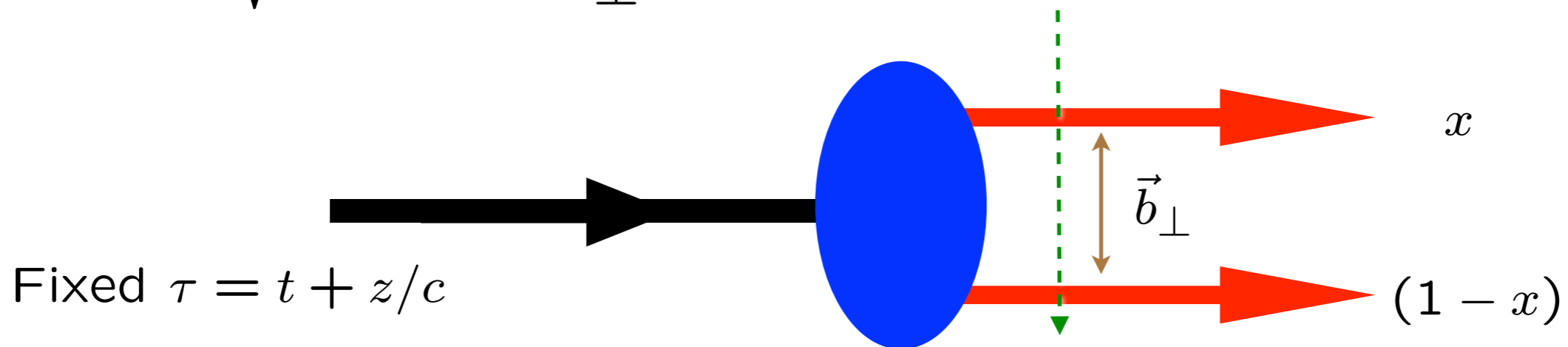
the solution for  $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$  !

**de Teramond, sjb**

*Identical to Polchinski-Strassler Convolution of AdS Amplitudes*

$LF(3+1) \longleftrightarrow AdS_5$ 

# Light-Front Holographic Dictionary

 $\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$ 
 $\zeta = \sqrt{x(1-x)b_\perp^2} \longleftrightarrow z$ 


$$\psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J - 2)^2$$

**Light-Front Holography:** Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion



$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

**Positive-sign dilaton**

• de Teramond, sjb

*AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:*

$$\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \Phi(z) = \mathcal{M}^2 \Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

*Derived from variation of Action for Dilaton-Modified AdS<sub>5</sub>*

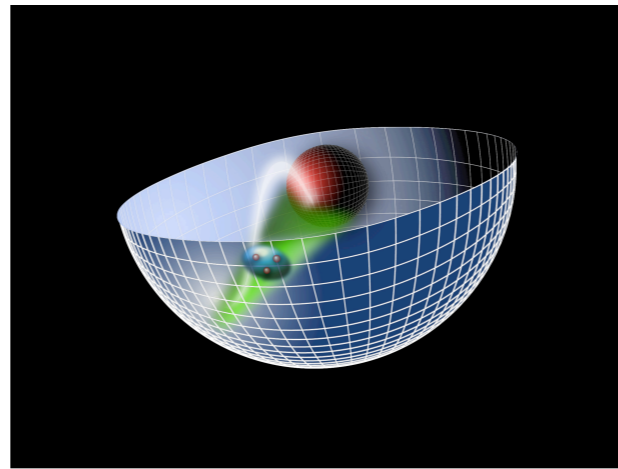
***Identical to Single-Variable Light-Front Bound State Equation in  $\zeta$ !***

$$z \quad \longleftrightarrow \quad \zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$

***Light-Front Holography***

*AdS/QCD  
Soft-Wall Model*

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)b_{\perp}^2$$

*Light-Front Holography*

$$\left[ -\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = M^2 \psi(\zeta)$$



***Light-Front Schrödinger Equation***

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

*Single variable  $\zeta$*

***Unique  
Confinement Potential!***  
*Conformal Symmetry  
of the action*

***Confinement scale:***

$$\kappa \simeq 0.5 \text{ GeV}$$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

***Scale can appear in Hamiltonian and EQM  
without affecting conformal invariance of action!***

*GeV units external to QCD: Only Ratios of Masses Determined*

# Massless pion!

## Meson Spectrum in Soft Wall Model

$$m_\pi = 0 \text{ if } m_q = 0$$

*Pion: Negative term for  $J=0$  cancels positive terms from LFKE and potential*



- Effective potential:  $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2(J - 1)$

- LF WE

$$\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2(J - 1) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions  $\langle \phi | \phi \rangle = \int d\zeta \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^L(\kappa^2 \zeta^2)$$

- Eigenvalues

$$\mathcal{M}_{n,J,L}^2 = 4\kappa^2 \left( n + \frac{J+L}{2} \right)$$

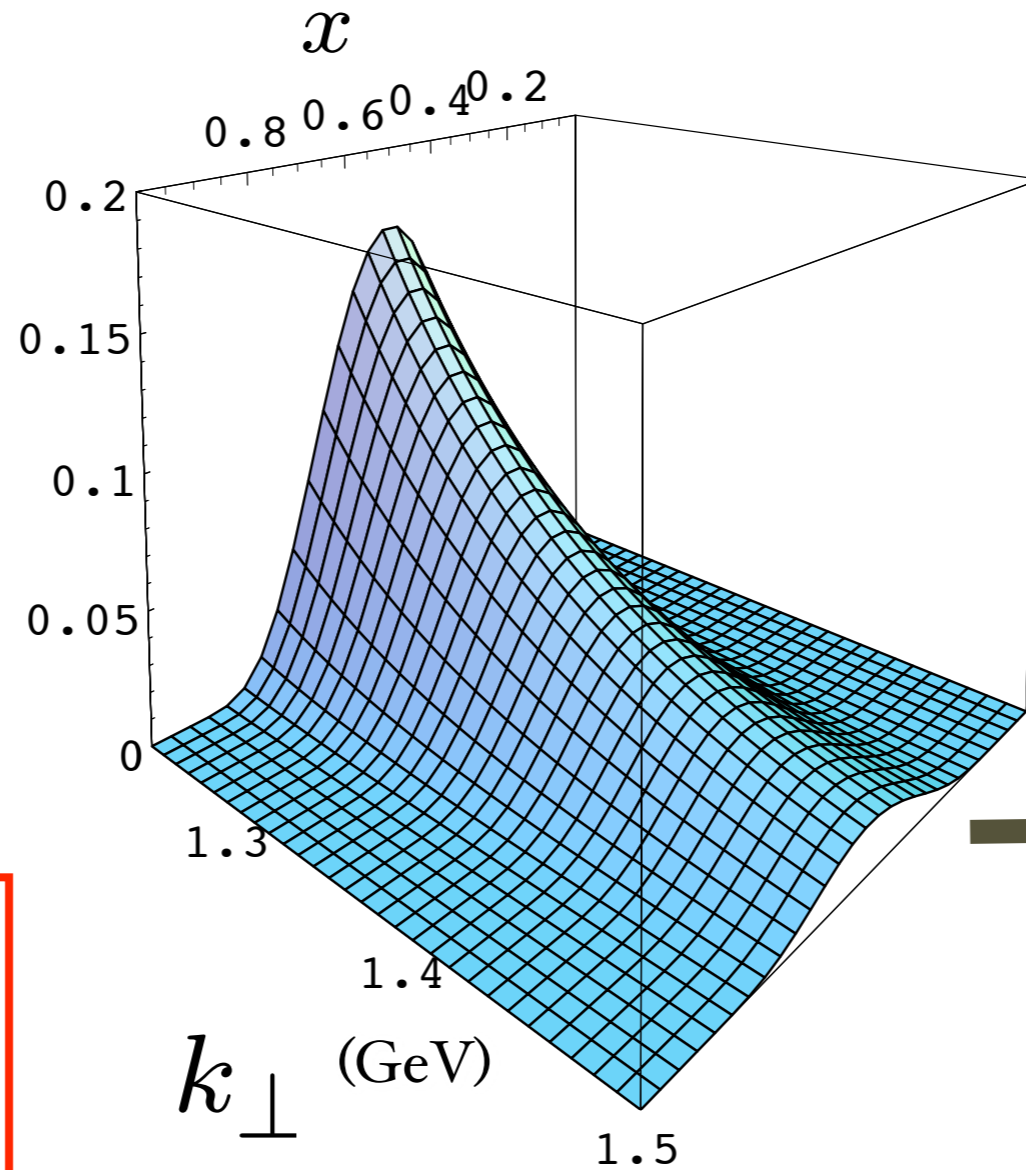
$$\vec{\zeta}^2 = \vec{b}_\perp^2 x(1-x)$$

**G. de Teramond, H. G. Dosch, sjb**

# Prediction from AdS/QCD: Meson LFWF

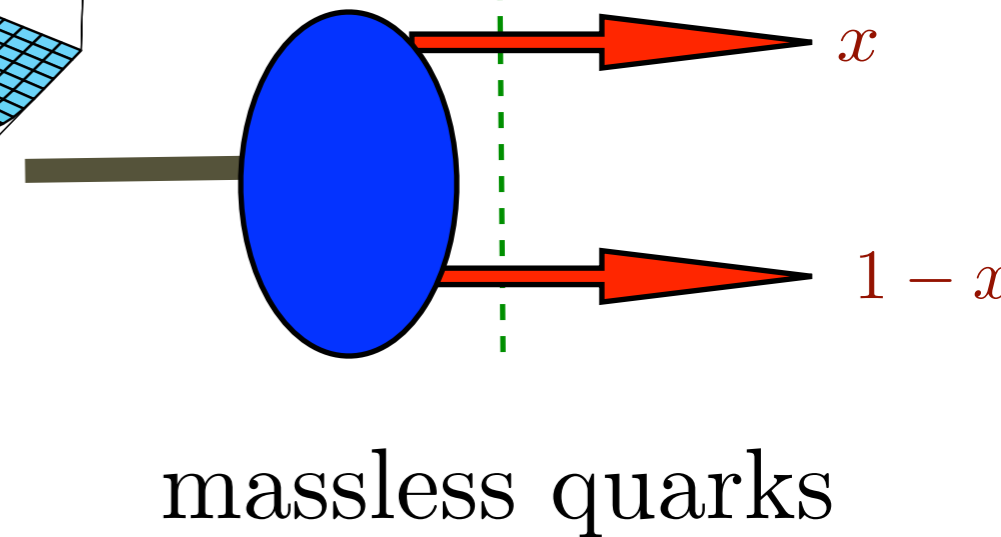
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_{\perp}^2)$$



de Teramond,  
Cao, sjb

“Soft Wall”  
model



**Note coupling**

$$k_{\perp}^2, x$$

$$\psi_M(x, k_{\perp}) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_{\perp}^2}{2\kappa^2 x(1-x)}}$$

$$\phi_{\pi}(x) = \frac{4}{\sqrt{3}\pi} f_{\pi} \sqrt{x(1-x)}$$

$$f_{\pi} = \sqrt{P_{q\bar{q}}} \frac{\sqrt{3}}{8} \kappa = 92.4 \text{ MeV}$$

**Same as DSE!** C. D. Roberts et al.

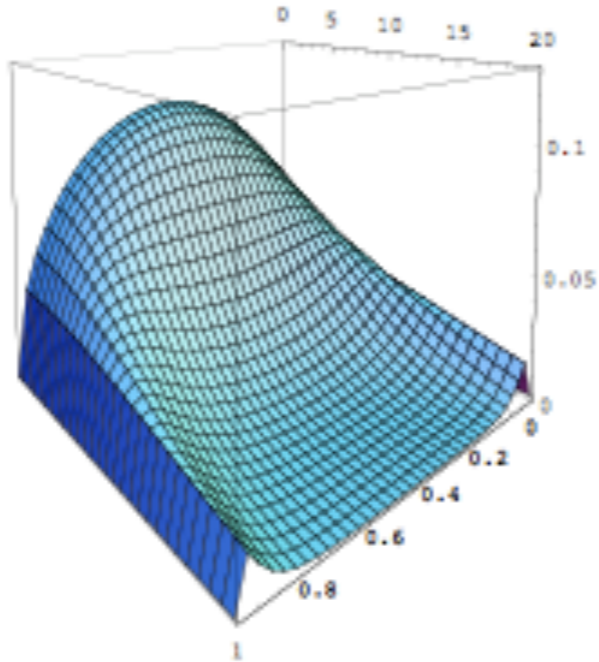
*Provides Connection of Confinement to Hadron Structure*



$$|\pi^+\rangle = |u\bar{d}\rangle$$

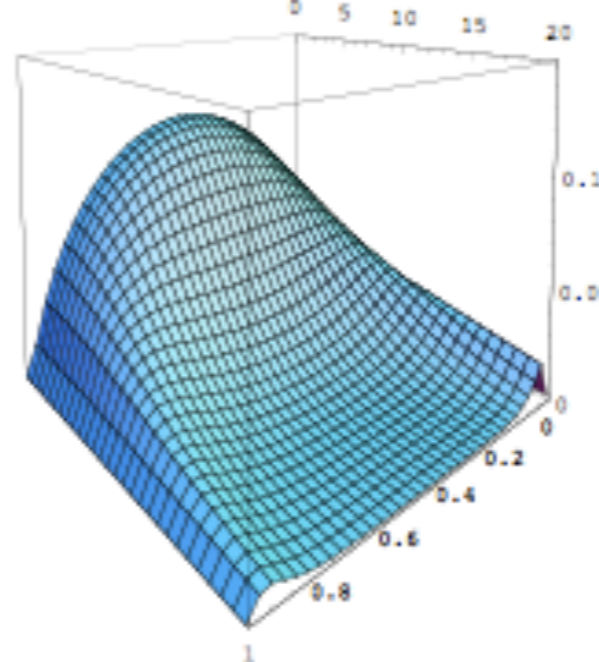
$$m_u = 2 \text{ MeV}$$

$$m_d = 5 \text{ MeV}$$



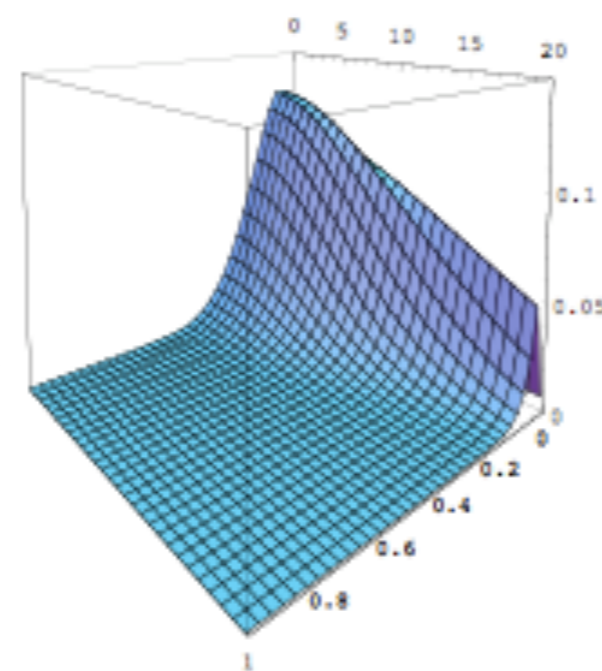
$$|K^+\rangle = |u\bar{s}\rangle$$

$$m_s = 95 \text{ MeV}$$

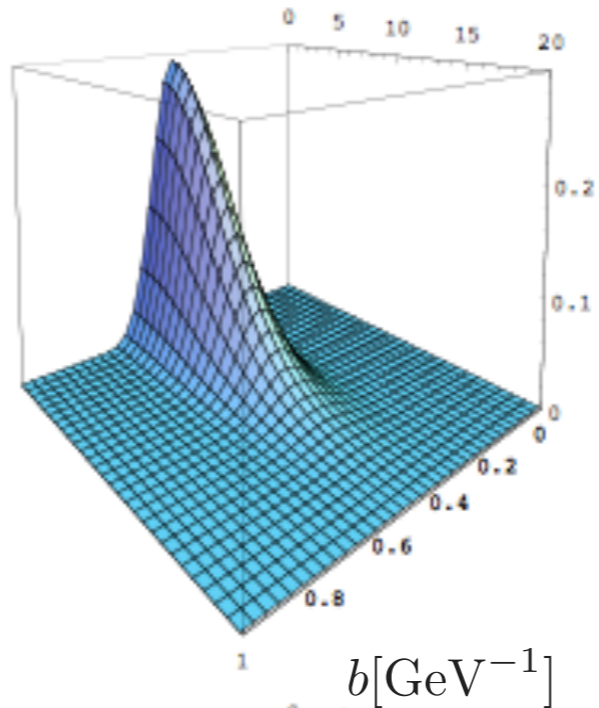


$$|D^+\rangle = |c\bar{d}\rangle$$

$$m_c = 1.25 \text{ GeV}$$

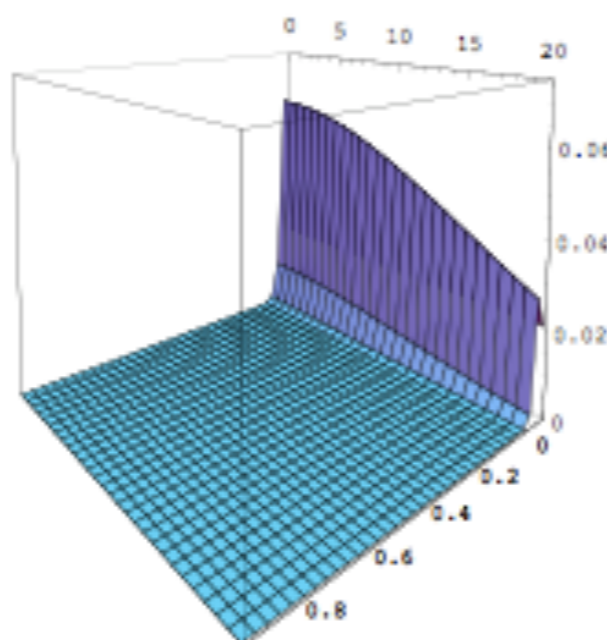


$$|\eta_c\rangle = |c\bar{c}\rangle$$



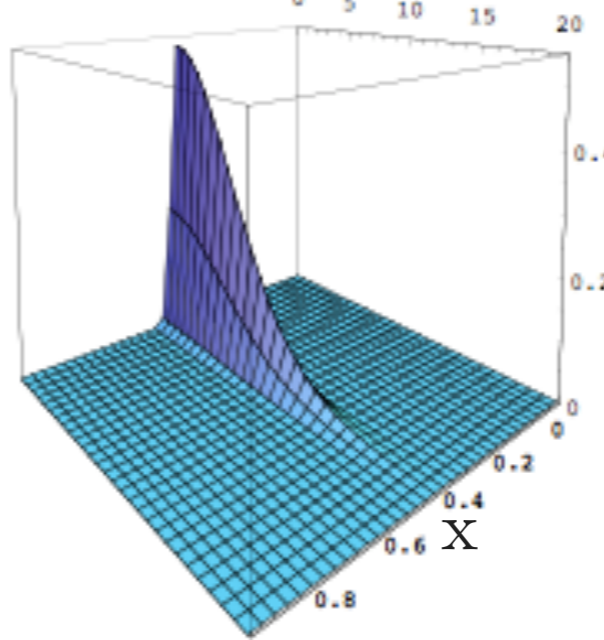
$$|B^+\rangle = |u\bar{b}\rangle$$

$$m_b = 4.2 \text{ GeV}$$

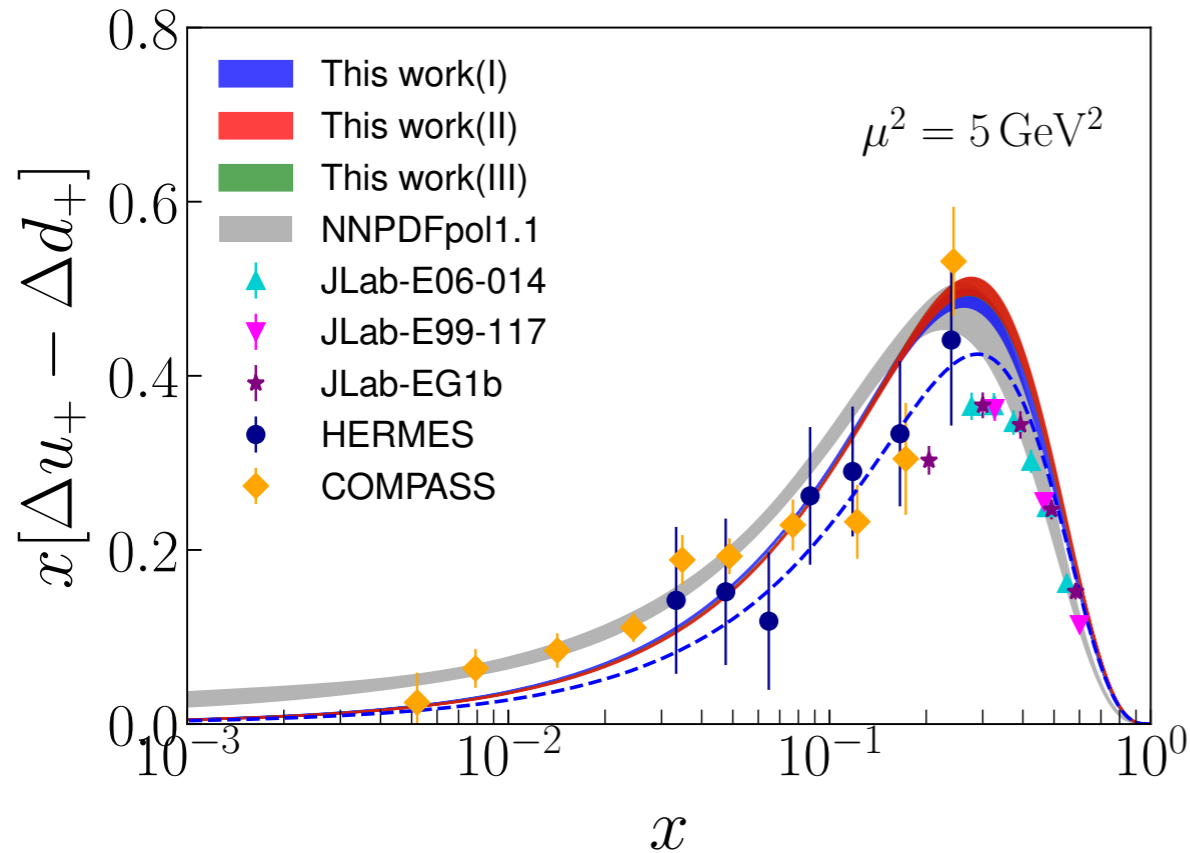


$$|\eta_b\rangle = |b\bar{b}\rangle$$

$$\kappa = 375 \text{ MeV}$$



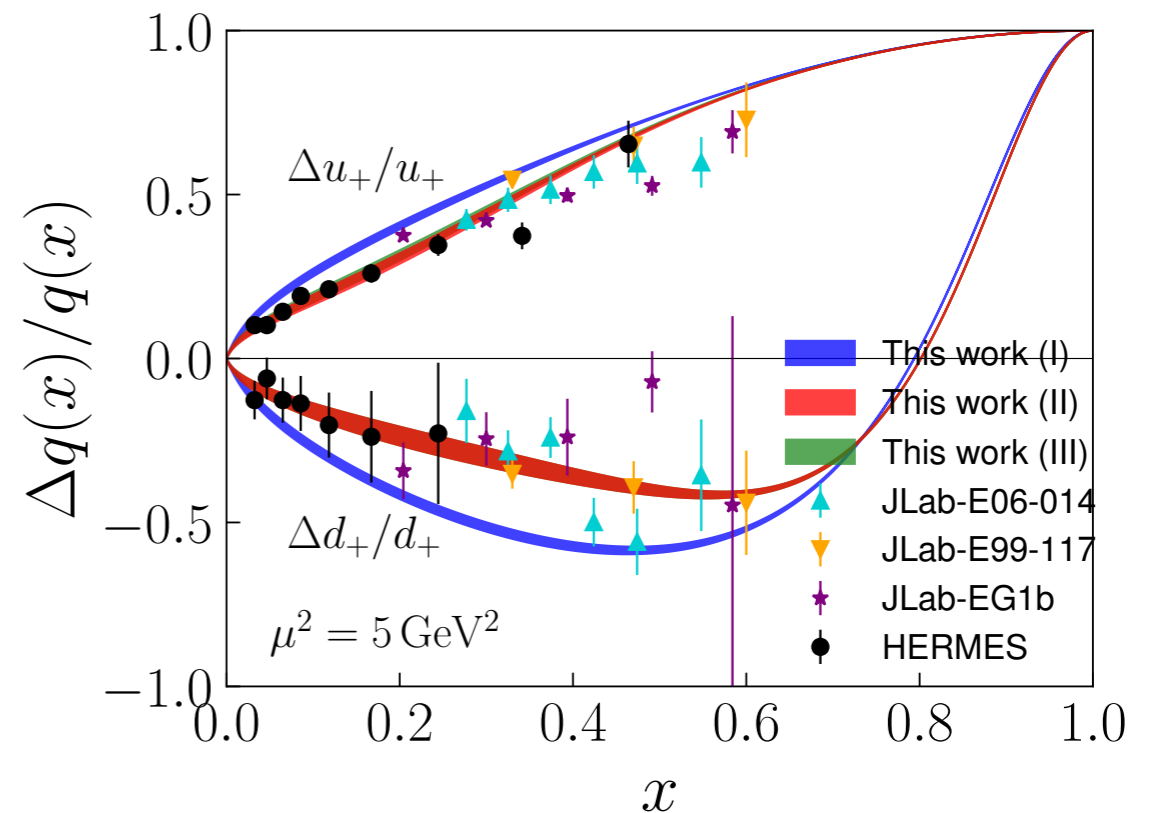
# Tianbo Liu, Raza Sabbir Sufian, Guy F. de T'era mond, Hans Gunter Dös ch, Alexandre Deur, s j b



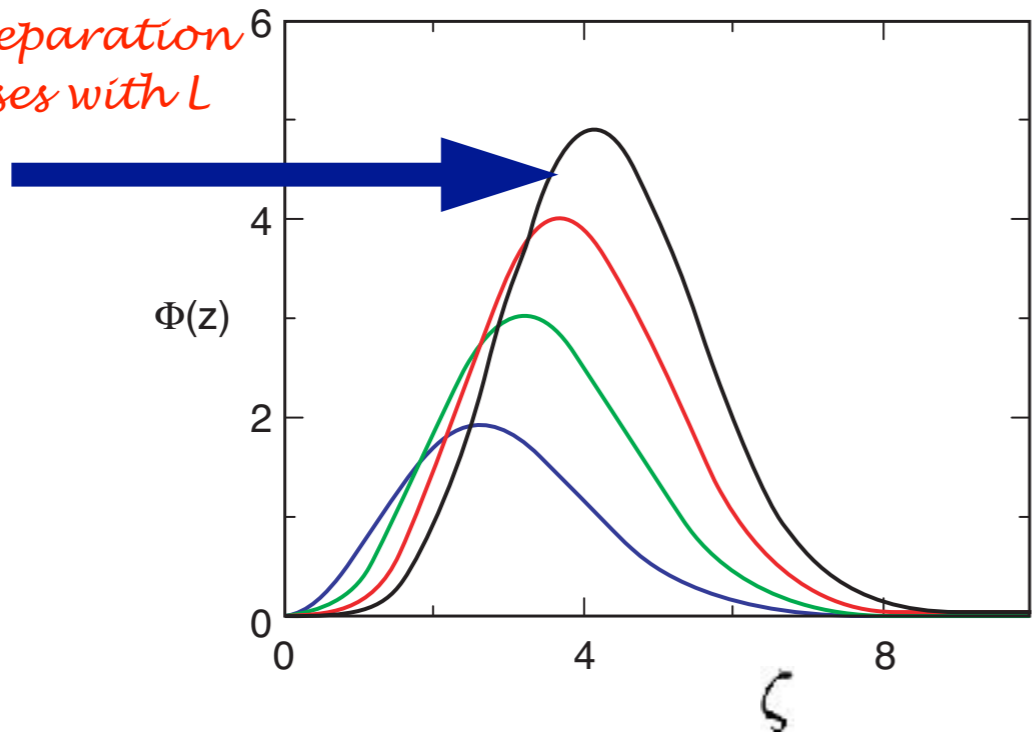
Polarized distributions for the isovector combination  $x[\Delta u_+(x) - \Delta d_+(x)]$

$$d_+(x) = d(x) + \bar{d}(x) \quad u_+(x) = u(x) + \bar{u}(x)$$

$$\Delta q(x) = q_{\uparrow}(x) - q_{\downarrow}(x)$$



Quark separation increases with  $L$



2-2007  
8721A21

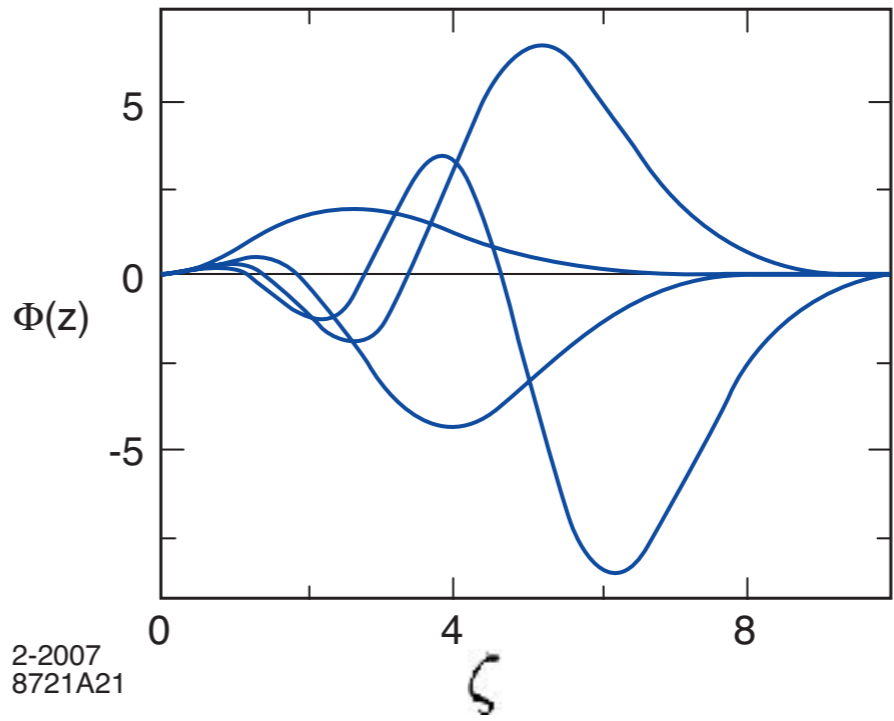
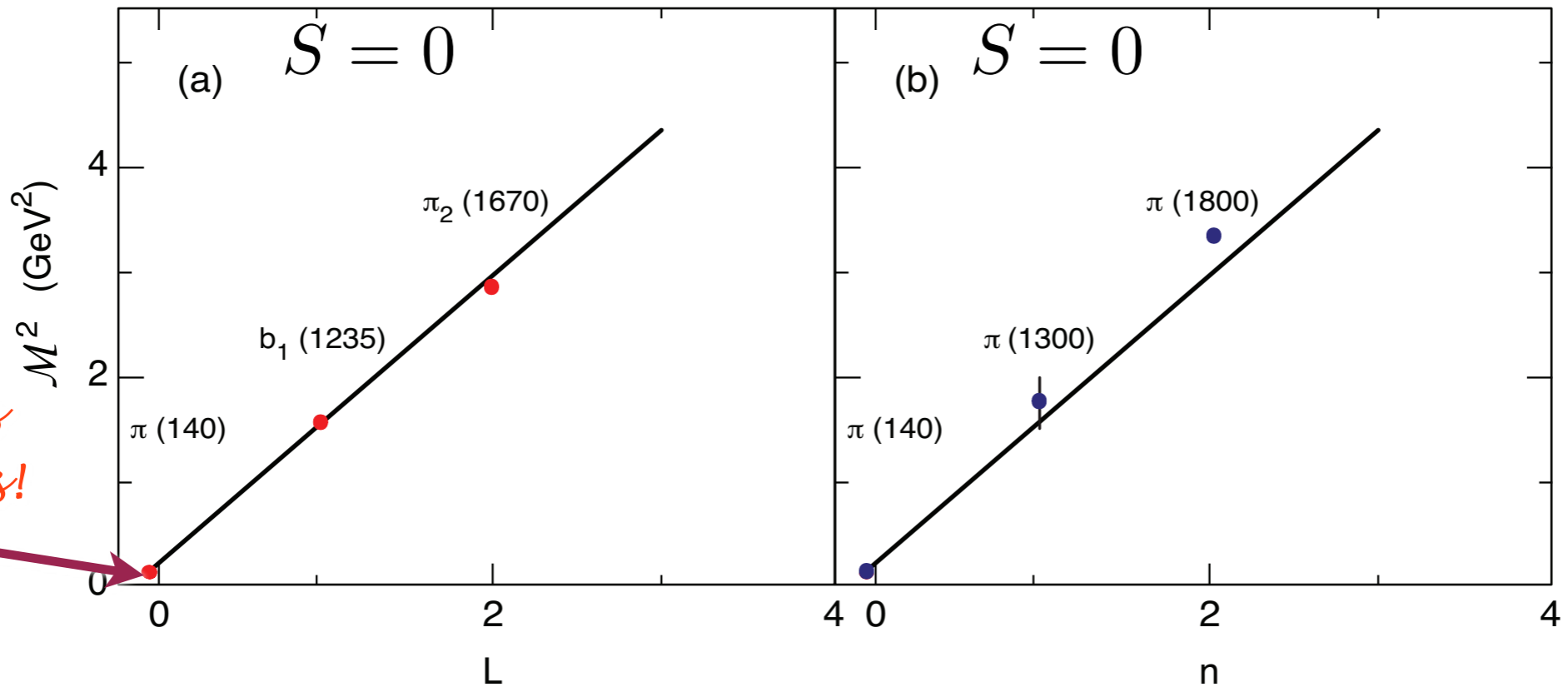


Fig: Orbital and radial AdS modes in the soft wall model for  $\kappa = 0.6$  GeV .

*Same slope in  $n$  and  $L$ !*

*Soft Wall Model*



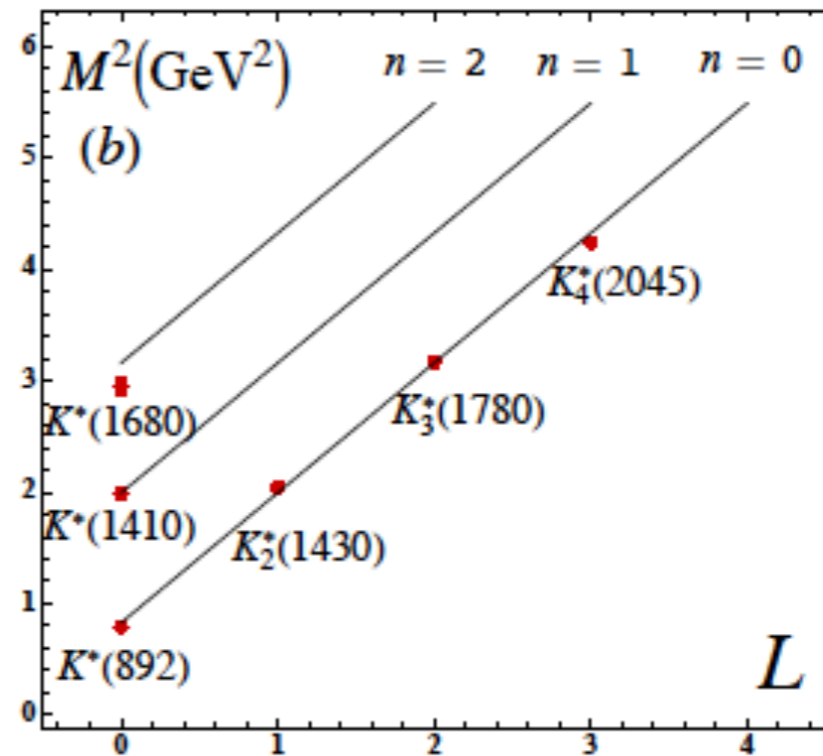
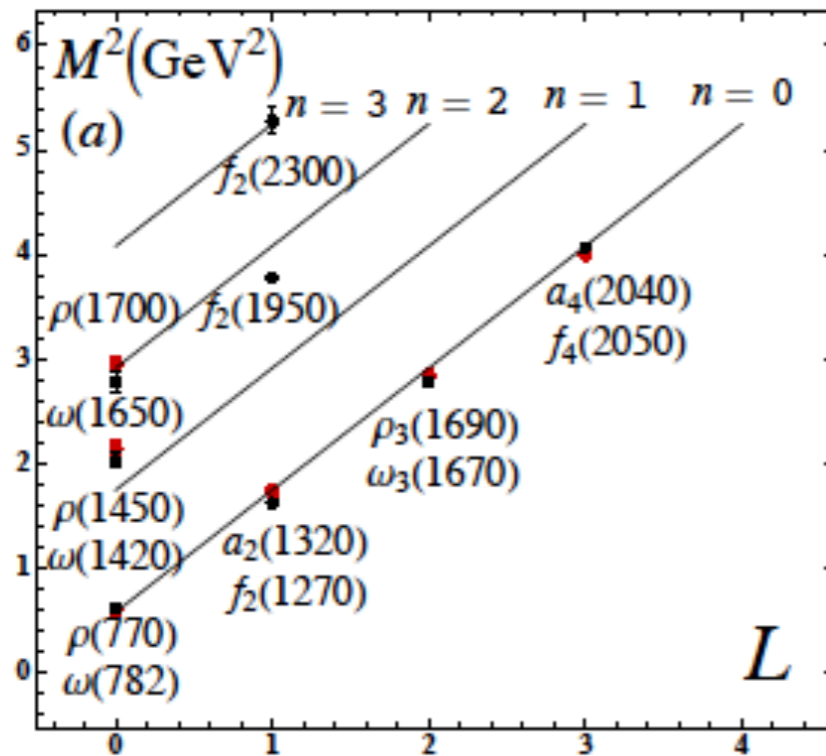
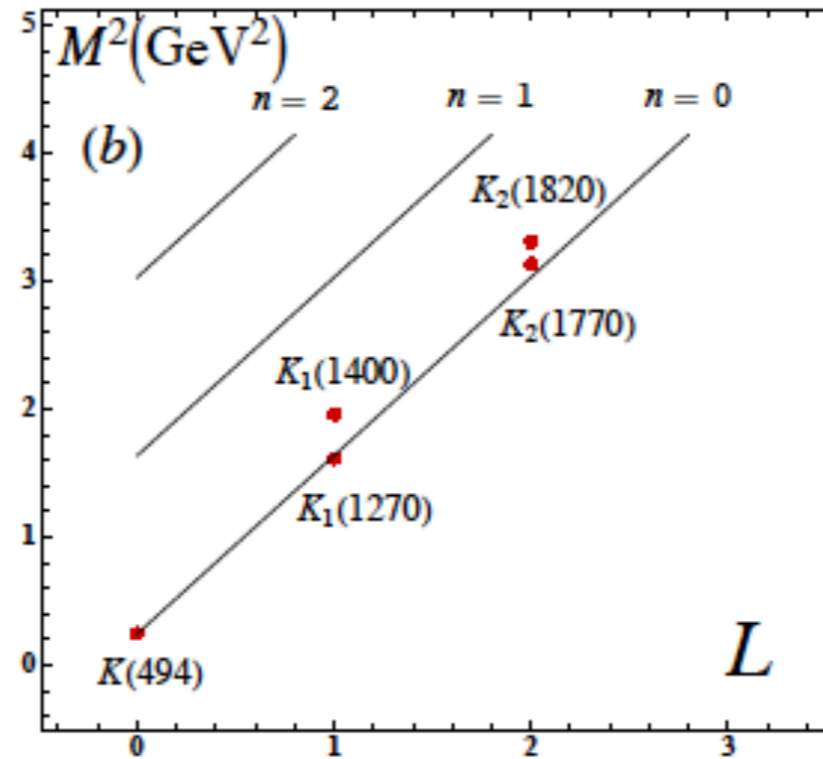
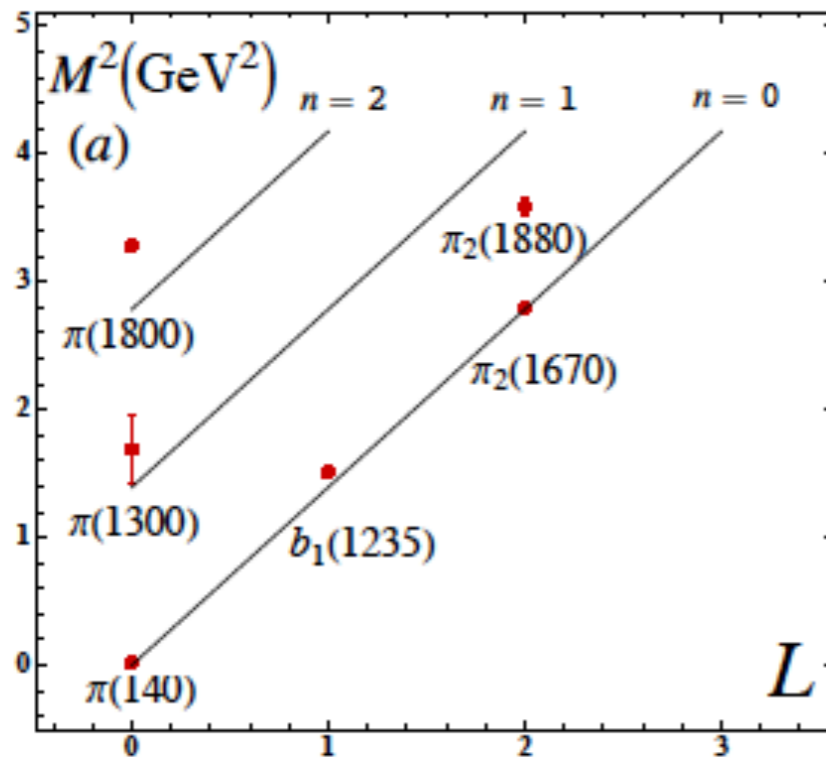
*Pion has zero mass!*

$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for  $\kappa = 0.6$  GeV.

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$

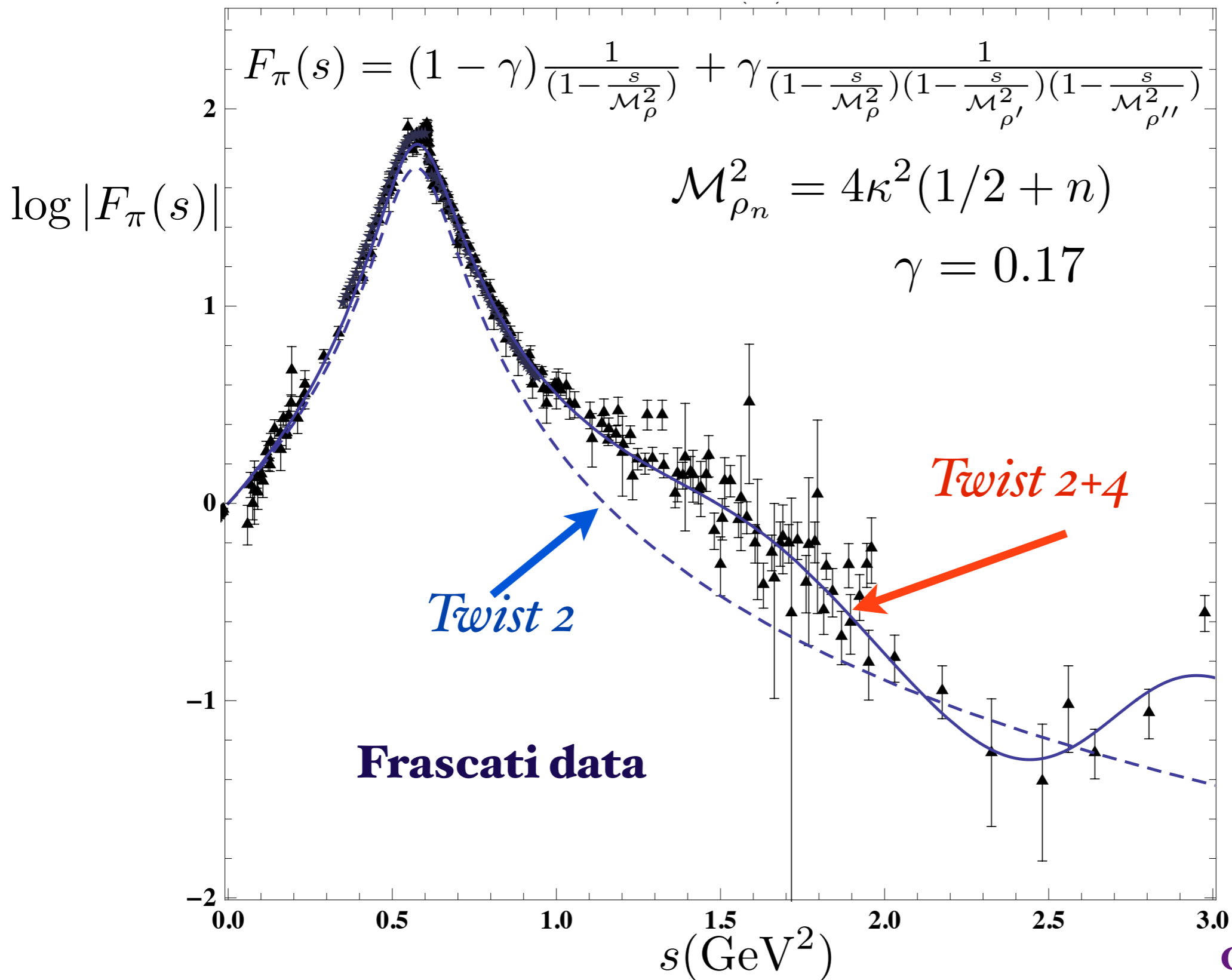
from LF Higgs mechanism



Effective mass from  $m(p^2)$



# Timelike Pion Form Factor from AdS/QCD and Light-Front Holography



**Prescription for  
Timelike poles :**

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

**14% four-quark  
probability**

# Connection to the Linear Instant-Form Potential

Linear instant nonrelativistic form  $V(r) = Cr$  for heavy quarks



Harmonic Oscillator  $U(\zeta) = \kappa^4 \zeta^2$  LF Potential for relativistic light quarks

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

# Remarkable Features of Light-Front Schrödinger Equation

## Dynamics + Spectroscopy!

- **Relativistic, frame-independent**
- **QCD scale appears - unique LF potential**
- **Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter**
- **Zero-mass pion for zero mass quarks!**
- **Regge slope same for n and L -- not usual HO**
- **Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry**
- **Phenomenology: LFWFs, Form factors, electroproduction**
- **Extension to heavy quarks**

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

*Stan Brodsky*  
*Bled Workshop*

Supersymmetric Features of Hadron Physics  
from Superconformal Algebra  
and Light-Front Holography

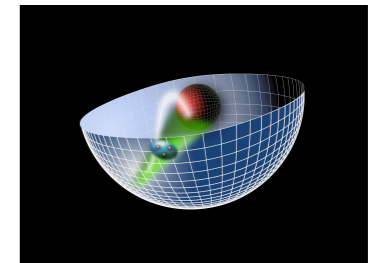


6 July 2021

# LFHQCD: Underlying Principles

- **Poincarè Invariance: Independent of the observer's Lorentz frame: Quantization at Fixed Light-Front Time  $\tau$**
- **Causality: Information within causal horizon: Light-Front**
- **Light-Front Holography:  $AdS_5 = LF (3+1)$**

$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_{\perp}^2 x(1-x)$$



- **Introduces Mass Scale  $\kappa$  while retaining the Conformal Invariance of the Action (dAFF)**
- **Unique Dilaton in  $AdS_5$ :  $e^{+\kappa^2 z^2}$**
- **Unique color-confining LF Potential  $U(\zeta^2) = \kappa^4 \zeta^2$**
- **Superconformal Algebra: Mass Degenerate 4-Plet:**

Meson  $q\bar{q} \leftrightarrow$  Baryon  $q[qq] \leftrightarrow$  Tetraquark  $[qq][\bar{q}\bar{q}]$

Superconformal Quantum Mechanics

$$\{\psi, \psi^+\} = 1 \quad B = \frac{1}{2}[\psi^+, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^+[-\partial_x + \frac{f}{x}], \quad Q^+ = \psi[\partial_x + \frac{f}{x}], \quad S = \psi^+ x, \quad S^+ = \psi x$$

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\{Q, S^+\} = f - B + 2iD, \quad \{Q^+, S\} = f - B - 2iD$$

generates conformal algebra

$$[H, D] = iH, \quad [H, K] = 2iD, \quad [K, D] = -iK$$

$$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$$



$$\left( -\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2(L_B + 1) + \frac{4L_B^2 - 1}{4\zeta^2} \right) \psi_J^+ = M^2 \psi_J^+ \quad \left. \vphantom{\left( -\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2(L_B + 1) + \frac{4L_B^2 - 1}{4\zeta^2} \right) \psi_J^+ = M^2 \psi_J^+} \right\}$$

$$\left( -\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2 L_B + \frac{4(L_B + 1)^2 - 1}{4\zeta^2} \right) \psi_J^- = M^2 \psi_J^- \quad \left. \vphantom{\left( -\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2 L_B + \frac{4(L_B + 1)^2 - 1}{4\zeta^2} \right) \psi_J^- = M^2 \psi_J^-} \right\}$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1) \quad \mathbf{S=1/2, P=+}$$

## Meson Equation

$$\lambda = \kappa^2$$

$$\left( -\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2(J - 1) + \frac{4L_M^2 - 1}{4\zeta^2} \right) \phi_J = M^2 \phi_J$$

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

$\mathbf{S=0, P=+}$   
*Same  $\kappa$ !*

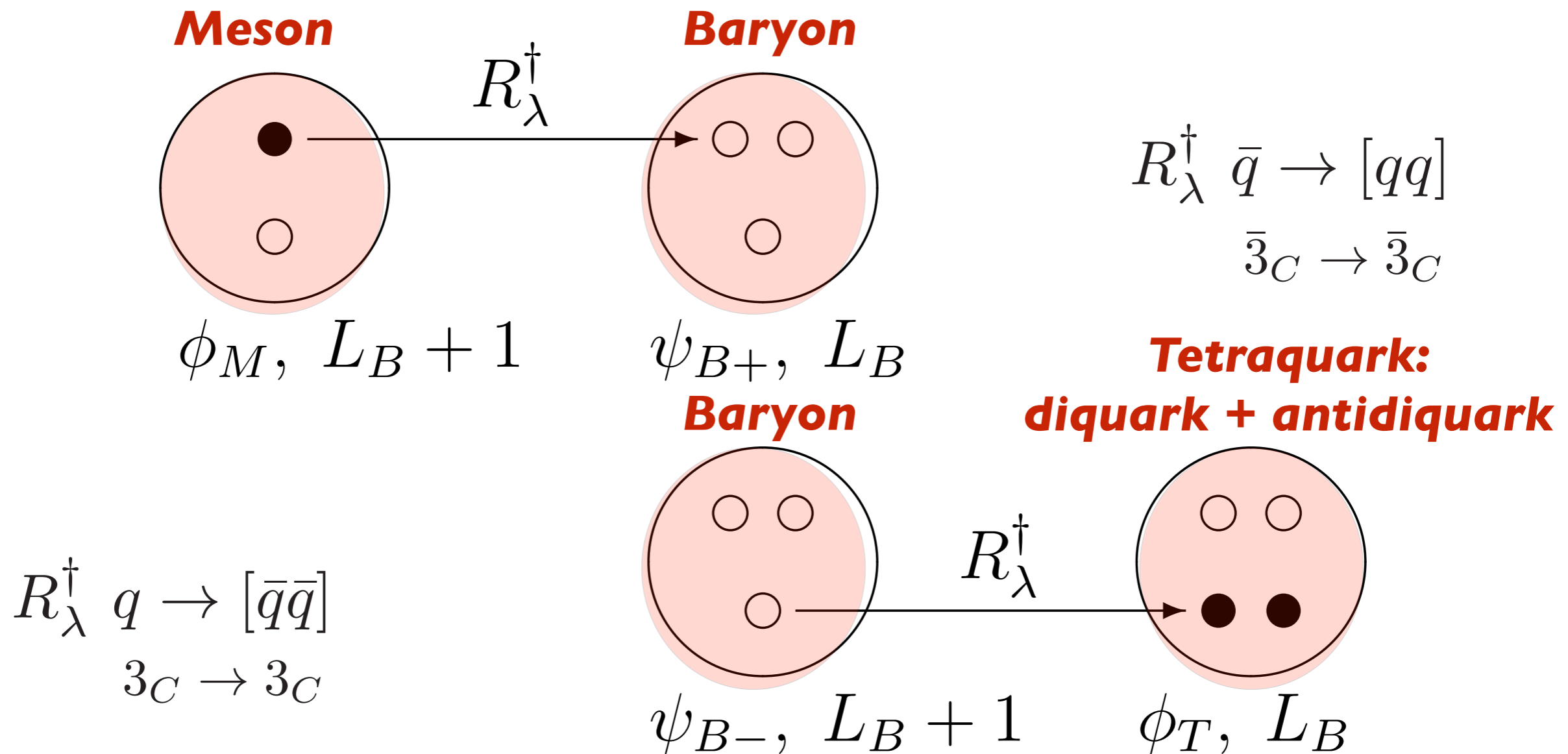
**$S=0, I=I$  Meson is superpartner of  $S=1/2, I=I$  Baryon**

**Meson-Baryon Degeneracy for  $L_M=L_B+1$**

# Superconformal Algebra

## 2X2 Hadronic Multiplets

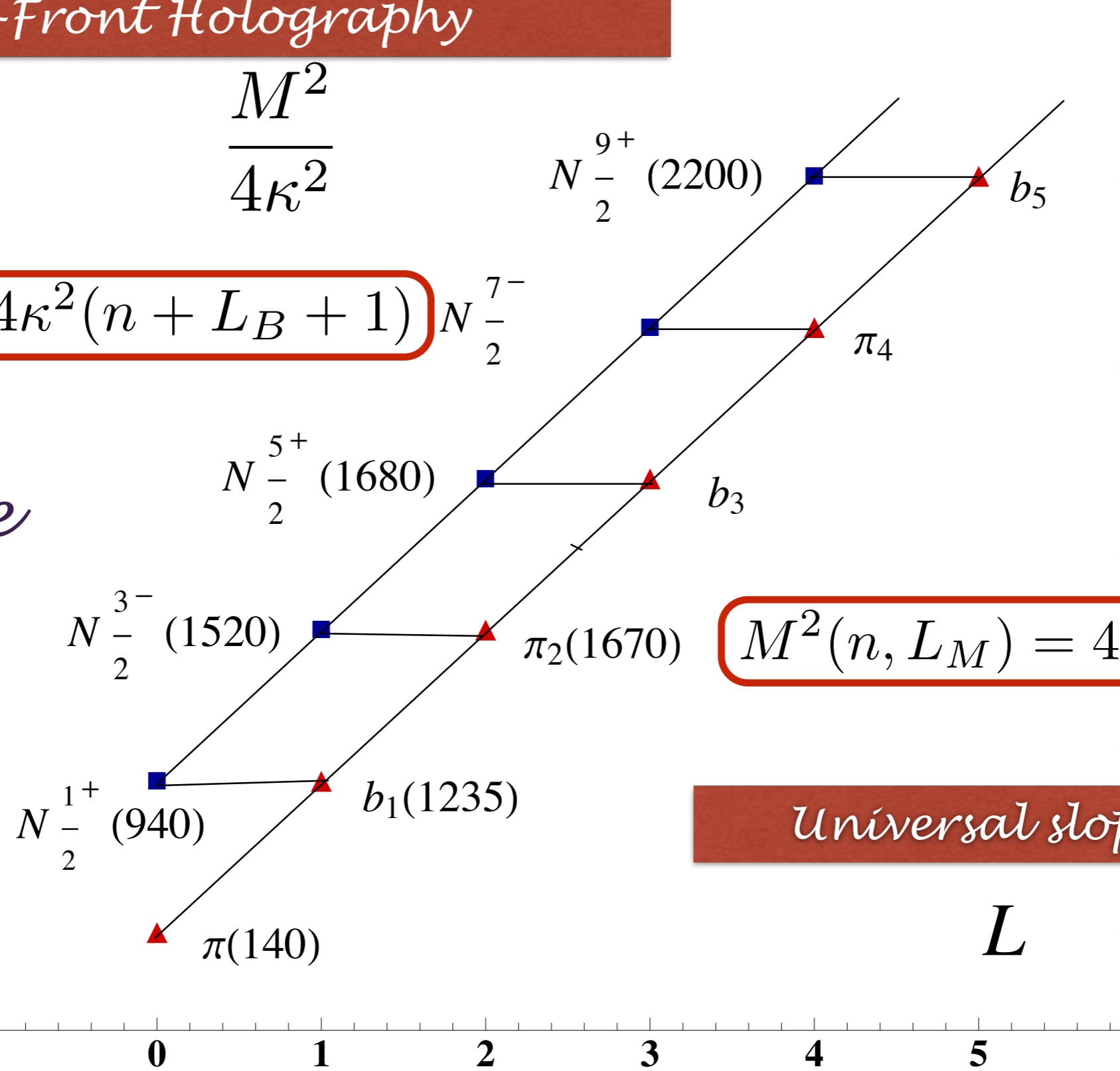
Bosons, Fermions with Equal Mass!



Proton:  $|u[ud]\rangle$  Quark + Scalar Diquark  
 Equal Weight:  $L=0, L=1$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

*Same slope*



$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

*Universal slopes in n, L*

$$\frac{M_{meson}^2}{M_{nucleon}^2} = \frac{n + L_M}{n + L_B + 1}$$

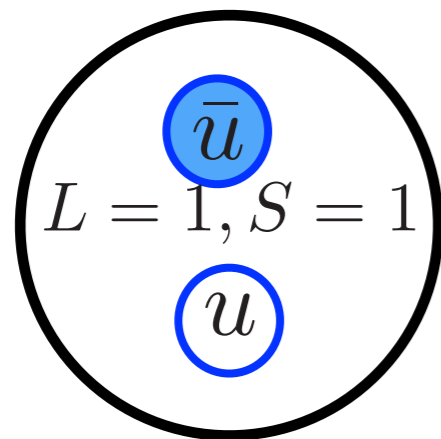
**Meson-Baryon  
Mass Degeneracy  
for L<sub>M</sub>=L<sub>B</sub>+1**

# Superconformal Algebra 4-Plet

$$R_\lambda^\dagger \begin{matrix} \bar{q} \rightarrow (qq) \\ \bar{3}_C \rightarrow \bar{3}_C \end{matrix} \quad S = 1$$

Vector ( ) + Scalar [ ] Diquarks

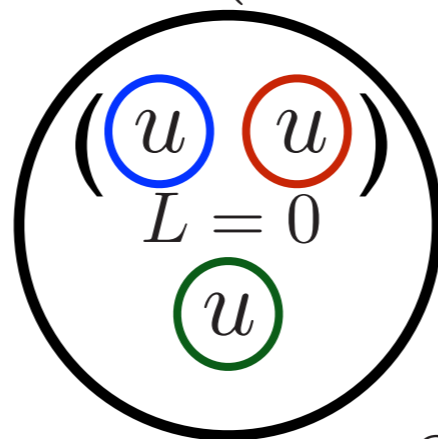
$f_2(1270)$



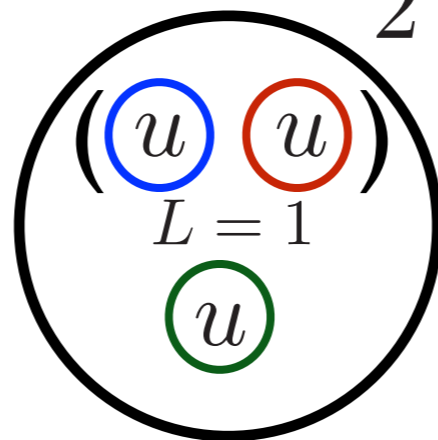
$$J^{PC} = 2^{++}$$

**Meson**

$\Delta^+(1232)$



$$J^P = \frac{3}{2}^+$$

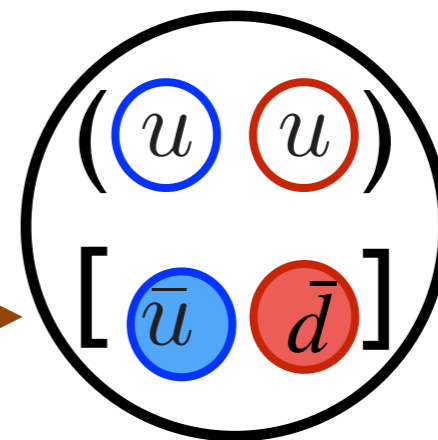


**Baryon**

**Tetraquark**

$$J^{PC} = 1^{++}$$

$a_1(1260)$



$$S = 0$$

$$L = 0$$

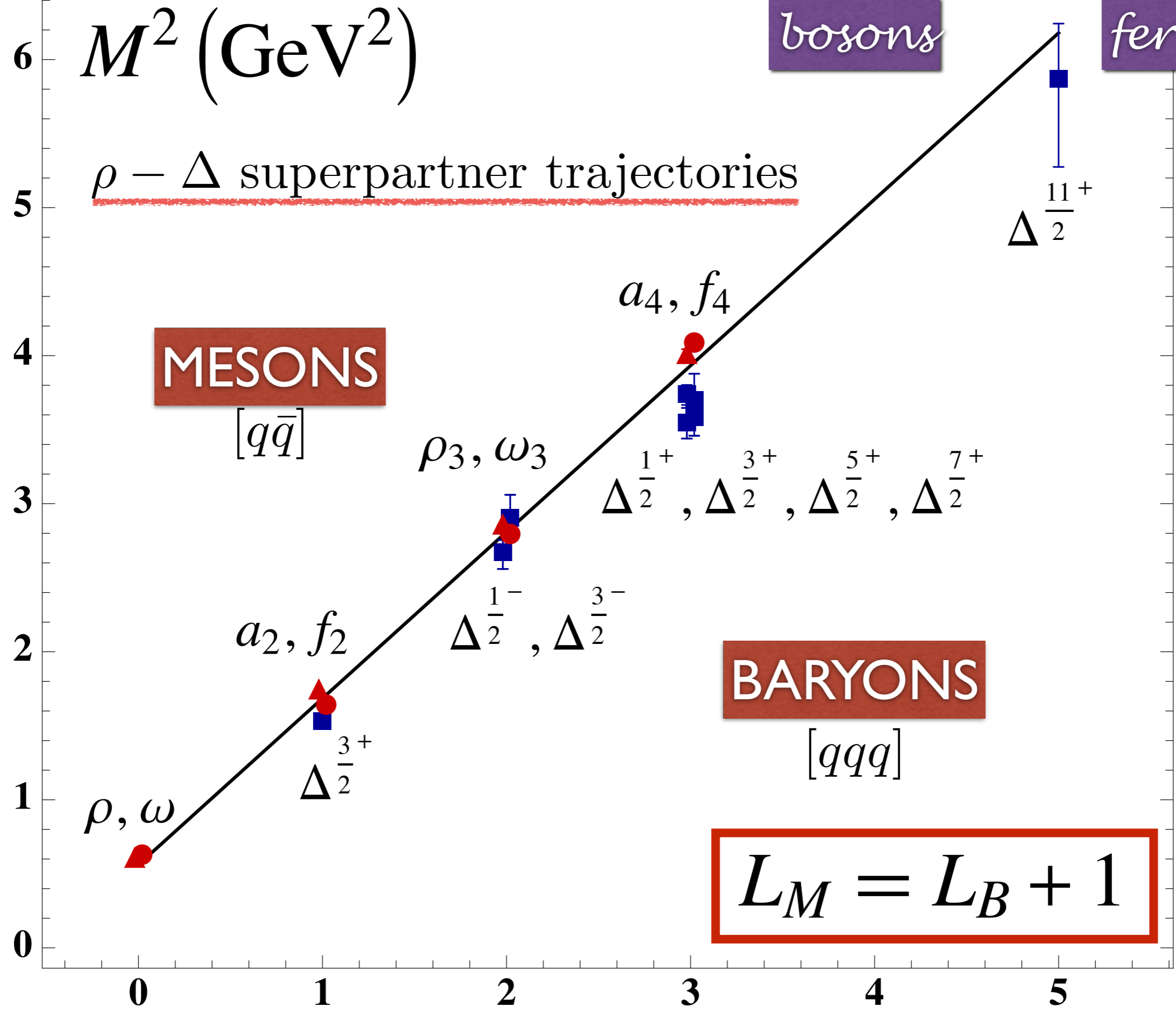
$$R_\lambda^\dagger \begin{matrix} q \rightarrow [\bar{q}\bar{q}] \\ 3_C \rightarrow 3_C \end{matrix}$$

$M^2$  (GeV<sup>2</sup>)

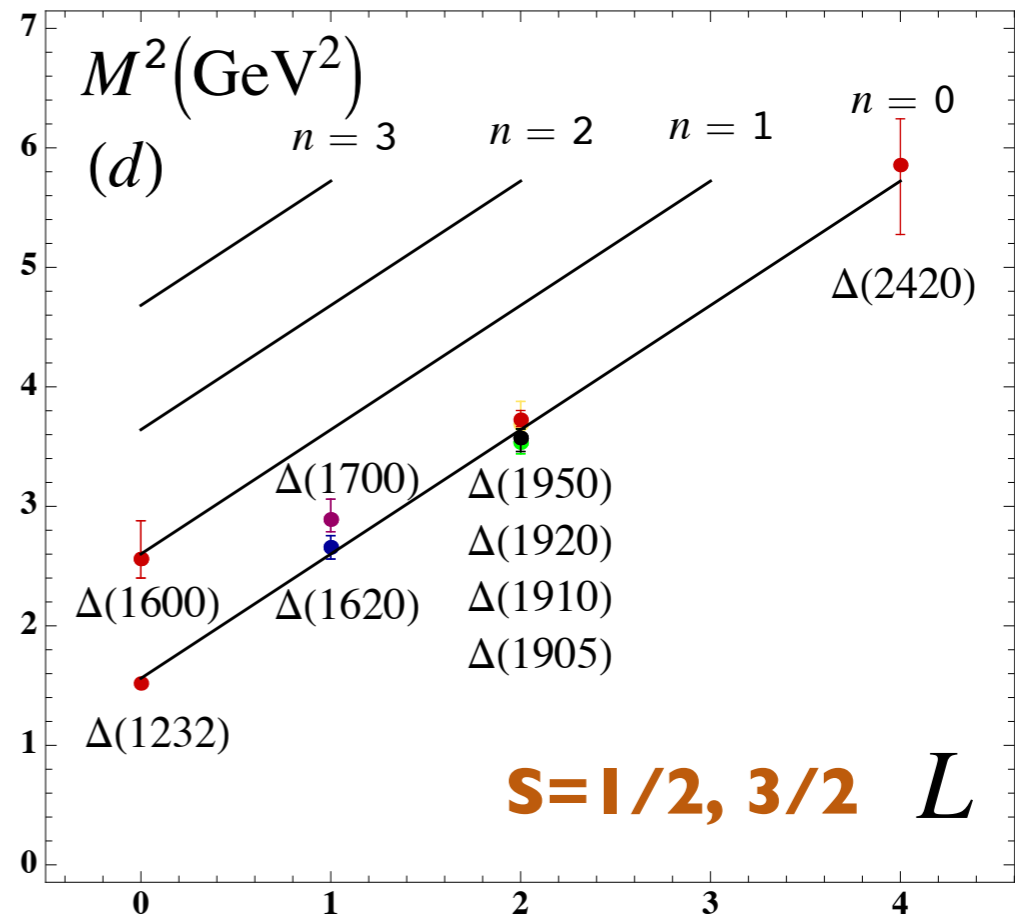
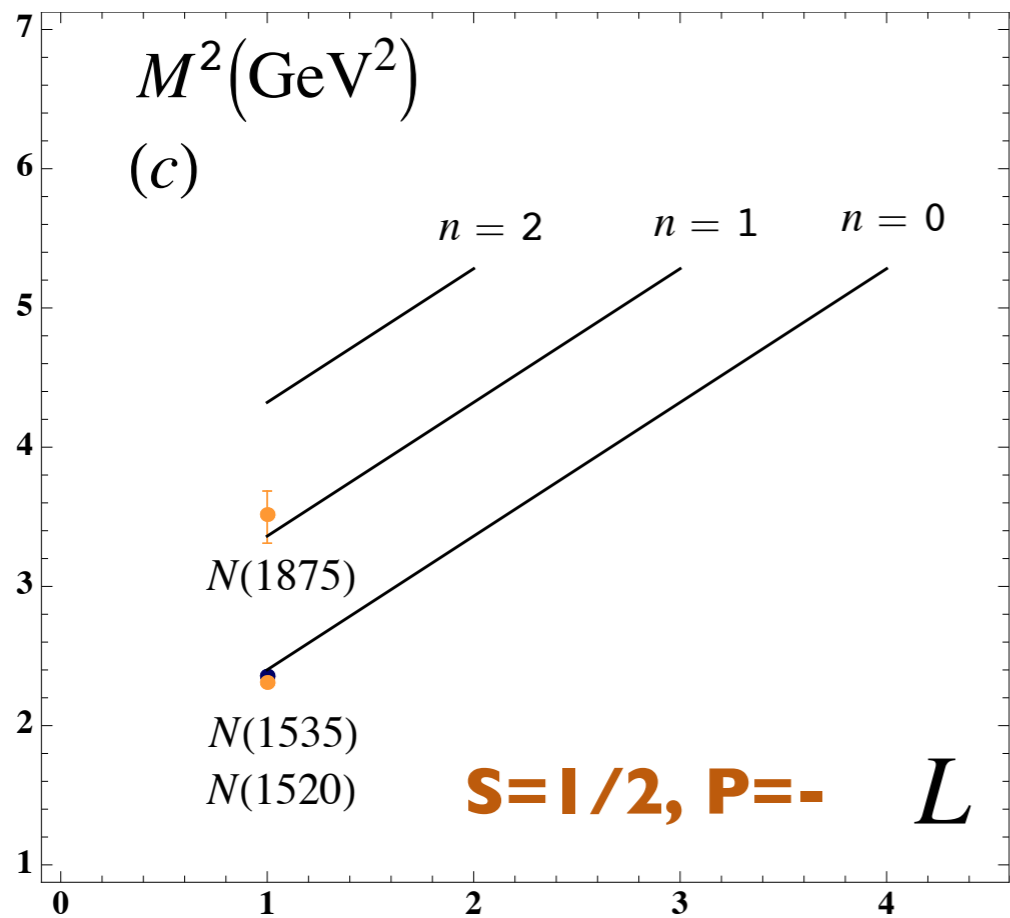
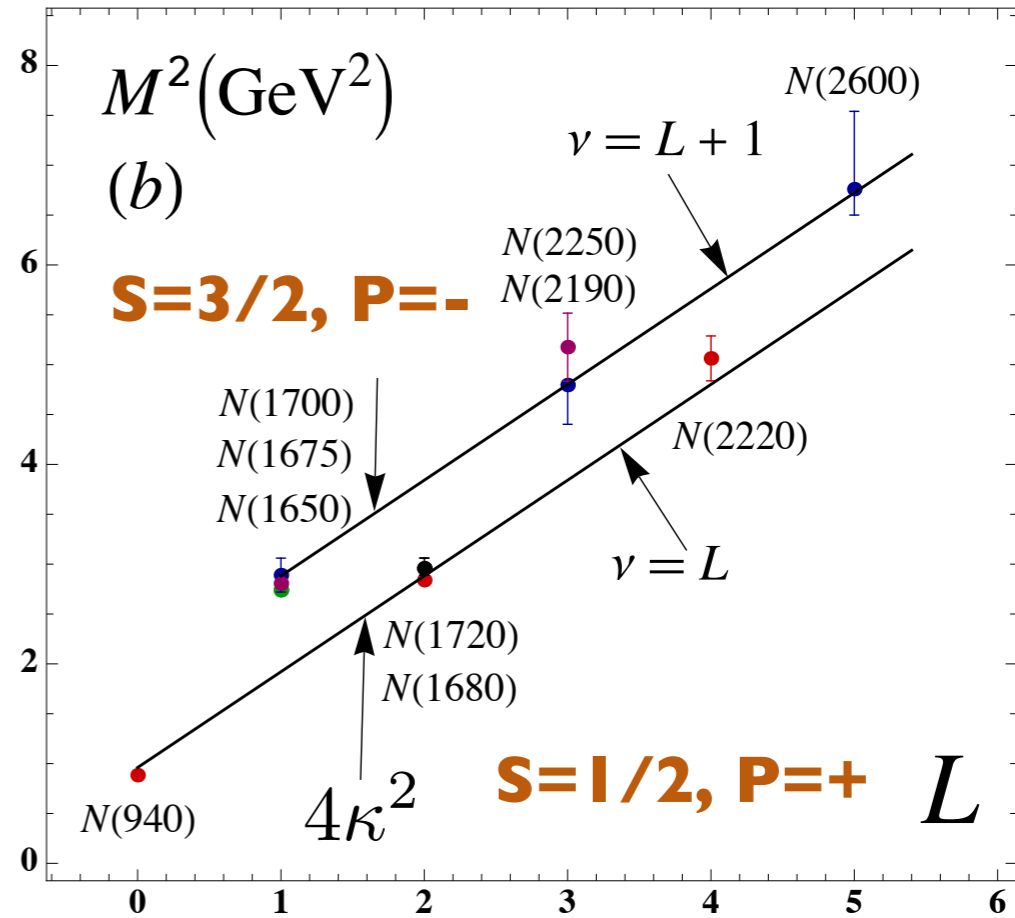
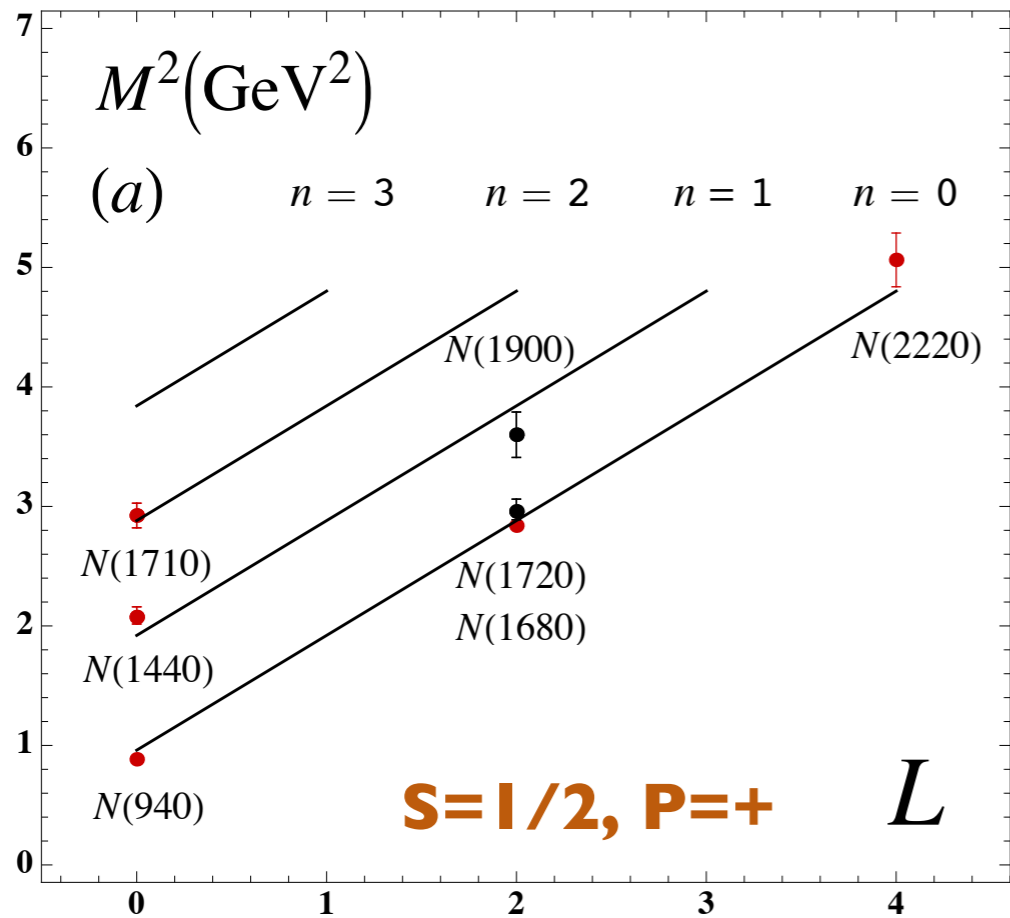
bosons

fermions

$\rho - \Delta$  superpartner trajectories







# Universal Hadronic Decomposition

$$\frac{\mathcal{M}_H^2}{\kappa^2} = (1 + 2n + L) + (1 + 2n + L) + (2L + 4S + 2B - 2)$$

- **Universal quark light-front kinetic energy**

$$\Delta\mathcal{M}_{LFKE}^2 = \kappa^2(1 + 2n + L)$$

- **Universal quark light-front potential energy**

$$\Delta\mathcal{M}_{LFPE}^2 = \kappa^2(1 + 2n + L)$$

- **Universal Constant Contribution from AdS and Superconformal Quantum Mechanics**

$$\Delta\mathcal{M}_{spin}^2 = 2\kappa^2(L + 2S + B - 1)$$

hyperfine spin-spin

**Equal:  
Virial  
Theorem**

Meson			Baryon			Tetraquark		
$q\text{-cont}$	$J^{P(C)}$	Name	$q\text{-cont}$	$J^P$	Name	$q\text{-cont}$	$J^{P(C)}$	Name
$\bar{q}q$	$0^{-+}$	$\pi(140)$	—	—	—	—	—	—
$\bar{q}q$	$1^{+-}$	$b_1(1235)$	$[ud]q$	$(1/2)^+$	$N(940)$	$[ud][\bar{u}\bar{d}]$	$0^{++}$	$f_0(980)$
$\bar{q}q$	$2^{-+}$	$\pi_2(1670)$	$[ud]q$	$(1/2)^-$	$N_{\frac{1}{2}}^-(1535)$	$[ud][\bar{u}\bar{d}]$	$1^{-+}$	$\pi_1(1400)$
				$(3/2)^-$	$N_{\frac{3}{2}}^-(1520)$			$\pi_1(1600)$
$\bar{q}q$	$1^{--}$	$\rho(770), \omega(780)$	—	—	—	—	—	—
$\bar{q}q$	$2^{++}$	$a_2(1320), f_2(1270)$	$[qq]q$	$(3/2)^+$	$\Delta(1232)$	$[qq][\bar{u}\bar{d}]$	$1^{++}$	$a_1(1260)$
$\bar{q}q$	$3^{--}$	$\rho_3(1690), \omega_3(1670)$	$[qq]q$	$(1/2)^-$	$\Delta_{\frac{1}{2}}^-(1620)$	$[qq][\bar{u}\bar{d}]$	$2^{--}$	$\rho_2(\sim 1700)?$
				$(3/2)^-$	$\Delta_{\frac{3}{2}}^-(1700)$			
$\bar{q}q$	$4^{++}$	$a_4(2040), f_4(2050)$	$[qq]q$	$(7/2)^+$	$\Delta_{\frac{7}{2}}^+(1950)$	$[qq][\bar{u}\bar{d}]$	$3^{++}$	$a_3(\sim 2070)?$
$\bar{q}s$	$0^{-+}$	$K(495)$	—	—	—	—	—	—
$\bar{q}s$	$1^{+-}$	$\bar{K}_1(1270)$	$[ud]s$	$(1/2)^+$	$\Lambda(1115)$	$[ud][\bar{s}\bar{q}]$	$0^{++}$	$K_0^*(1430)$
$\bar{q}s$	$2^{-+}$	$K_2(1770)$	$[ud]s$	$(1/2)^-$	$\Lambda(1405)$	$[ud][\bar{s}\bar{q}]$	$1^{-+}$	$K_1^*(\sim 1700)?$
				$(3/2)^-$	$\Lambda(1520)$			
$\bar{s}q$	$0^{-+}$	$K(495)$	—	—	—	—	—	—
$\bar{s}q$	$1^{+-}$	$K_1(1270)$	$[sq]q$	$(1/2)^+$	$\Sigma(1190)$	$[sq][\bar{s}\bar{q}]$	$0^{++}$	$a_0(980)$ $f_0(980)$
$\bar{s}q$	$1^{-+}$	$K^*(890)$	—	—	—	—	—	—
$\bar{s}q$	$2^{++}$	$K_2^*(1430)$	$[sq]q$	$(3/2)^+$	$\Sigma(1385)$	$[sq][\bar{q}\bar{q}]$	$1^{++}$	$K_1(1400)$
$\bar{s}q$	$3^{-+}$	$K_3^*(1780)$	$[sq]q$	$(3/2)^-$	$\Sigma(1670)$	$[sq][\bar{q}\bar{q}]$	$2^{-+}$	$K_2(\sim 1700)?$
$\bar{s}q$	$4^{++}$	$K_4^*(2045)$	$[sq]q$	$(7/2)^+$	$\Sigma(2030)$	$[sq][\bar{q}\bar{q}]$	$3^{++}$	$K_3(\sim 2070)?$
$\bar{s}s$	$0^{-+}$	$\eta(550)$	—	—	—	—	—	—
$\bar{s}s$	$1^{+-}$	$h_1(1170)$	$[sq]s$	$(1/2)^+$	$\Xi(1320)$	$[sq][\bar{s}\bar{q}]$	$0^{++}$	$f_0(1370)$ $a_0(1450)$
$\bar{s}s$	$2^{-+}$	$\eta_2(1645)$	$[sq]s$	$(?)^?$	$\Xi(1690)$	$[sq][\bar{s}\bar{q}]$	$1^{-+}$	$\Phi'(1750)?$
$\bar{s}s$	$1^{--}$	$\Phi(1020)$	—	—	—	—	—	—
$\bar{s}s$	$2^{++}$	$f_2'(1525)$	$[sq]s$	$(3/2)^+$	$\Xi^*(1530)$	$[sq][\bar{s}\bar{q}]$	$1^{++}$	$f_1(1420)$
$\bar{s}s$	$3^{--}$	$\Phi_3(1850)$	$[sq]s$	$(3/2)^-$	$\Xi(1820)$	$[sq][\bar{s}\bar{q}]$	$2^{--}$	$\Phi_2(\sim 1800)?$
$\bar{s}s$	$2^{++}$	$f_2(1950)$	$[ss]s$	$(3/2)^+$	$\Omega(1672)$	$[ss][\bar{s}\bar{q}]$	$1^{++}$	$K_1(\sim 1700)?$

Meson

Baryon

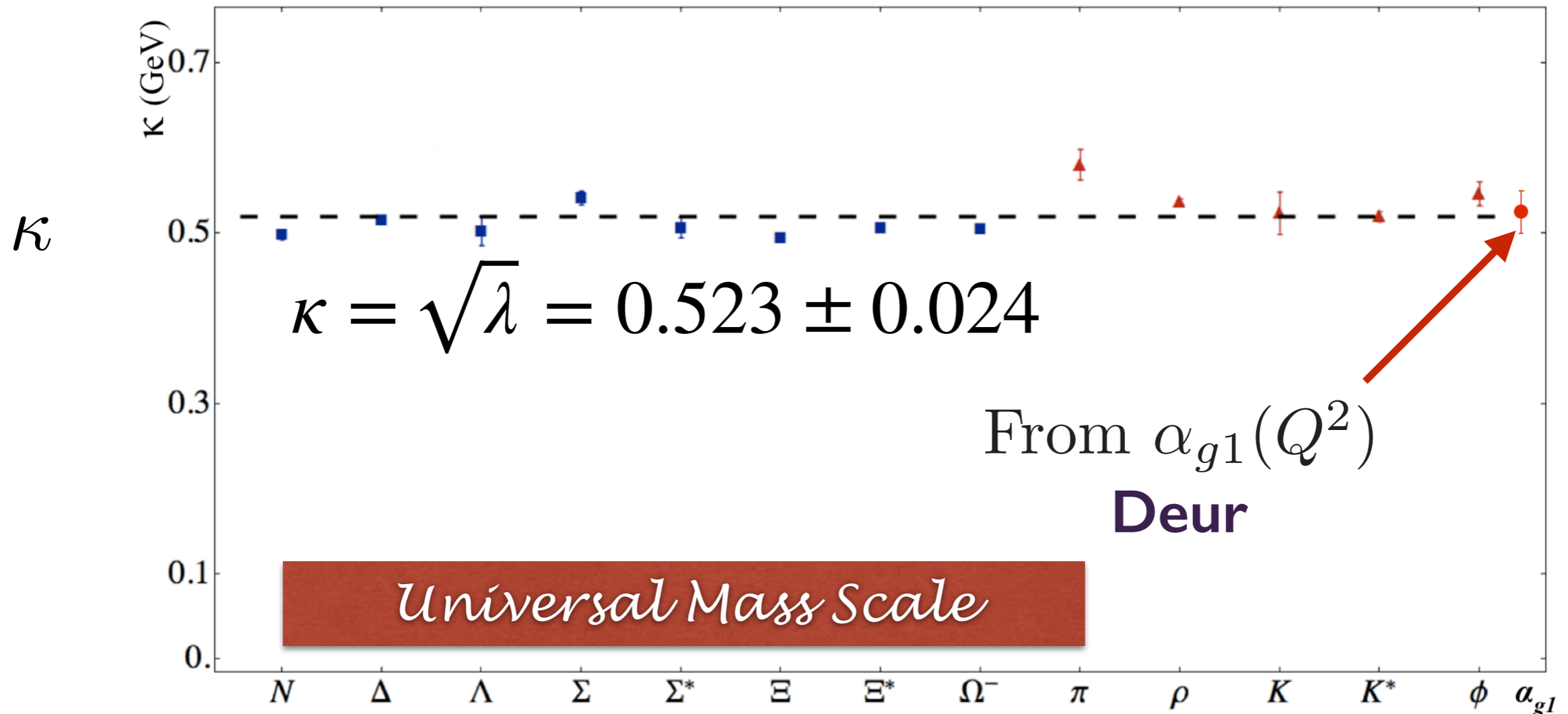
Tetraquark

**New Organization of the Hadron Spectrum**

M. Nielsen,  
sjb

$$\lambda = \kappa^2$$

$$m_u = m_d = 46 \text{ MeV}, m_s = 357 \text{ MeV}$$



**Fit to the slope of Regge trajectories,  
including radial excitations**

**Same Regge Slope for Meson, Baryons:  
Supersymmetric feature of hadron physics**



- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{3/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^{L+1}(\kappa^2 \zeta^2)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L} \frac{1}{\sqrt{n+L+2}} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{5/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^{L+2}(\kappa^2 \zeta^2)$$

- Normalization

$$\int d\zeta \psi_+^2(\zeta) = \int d\zeta \psi_-^2(\zeta) = 1$$

- Eigenvalues

$$\int_0^\infty d\zeta \int_0^1 dx \psi_+^2(\zeta^2, x) = \int_0^\infty d\zeta \int_0^1 dx \psi_-^2(\zeta^2, x) = \frac{1}{2}$$

*Quark Chiral  
Symmetry of  
Eigenstate!*

**Nucleon: Equal Probability for L=0, 1**

$$J^z = +1/2 : \frac{1}{\sqrt{2}} [ |S_q^z = +1/2, L^z = 0\rangle + |S_q^z = -1/2, L^z = +1\rangle ]$$

*Nucleon spin carried by quark orbital angular momentum*

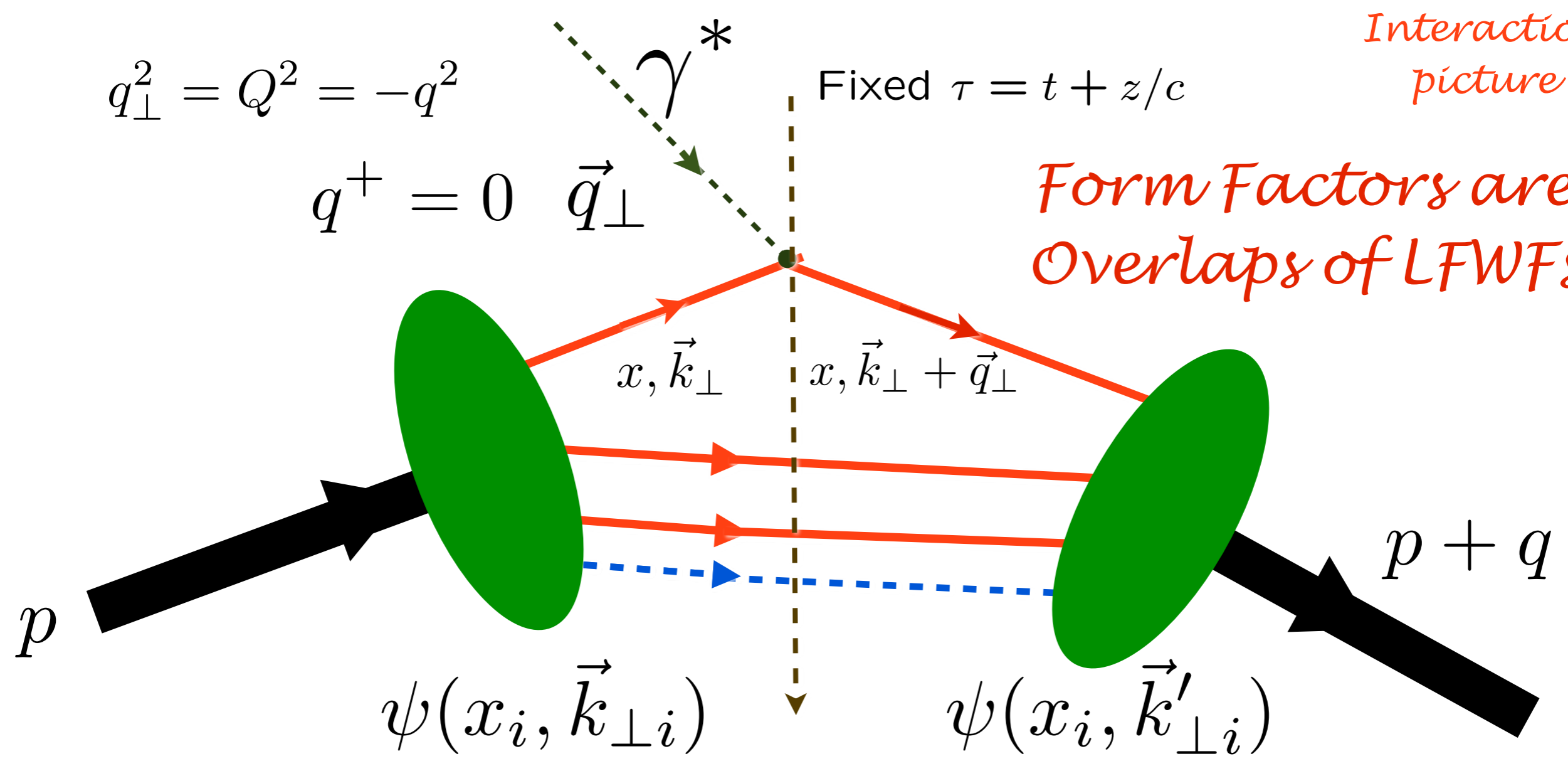


$$\langle p + q | j^+(0) | p \rangle = 2p^+ F(q^2)$$

**Front Form**

*Interaction picture*

*Form Factors are Overlaps of LFWFs*



$$q_{\perp}^2 = Q^2 = -q^2$$

$$q^+ = 0 \quad \vec{q}_{\perp}$$

Fixed  $\tau = t + z/c$

$$x, \vec{k}_{\perp}$$

$$x, \vec{k}_{\perp} + \vec{q}_{\perp}$$

$$\psi(x_i, \vec{k}_{\perp i})$$

$$\psi(x_i, \vec{k}'_{\perp i})$$

*struck*  $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_{\perp}$

*spectators*  $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_{\perp}$

**Drell & Yan, West  
Exact LF formula!**

Drell, sjb

# Exact LF Formula for Pauli Form Factor

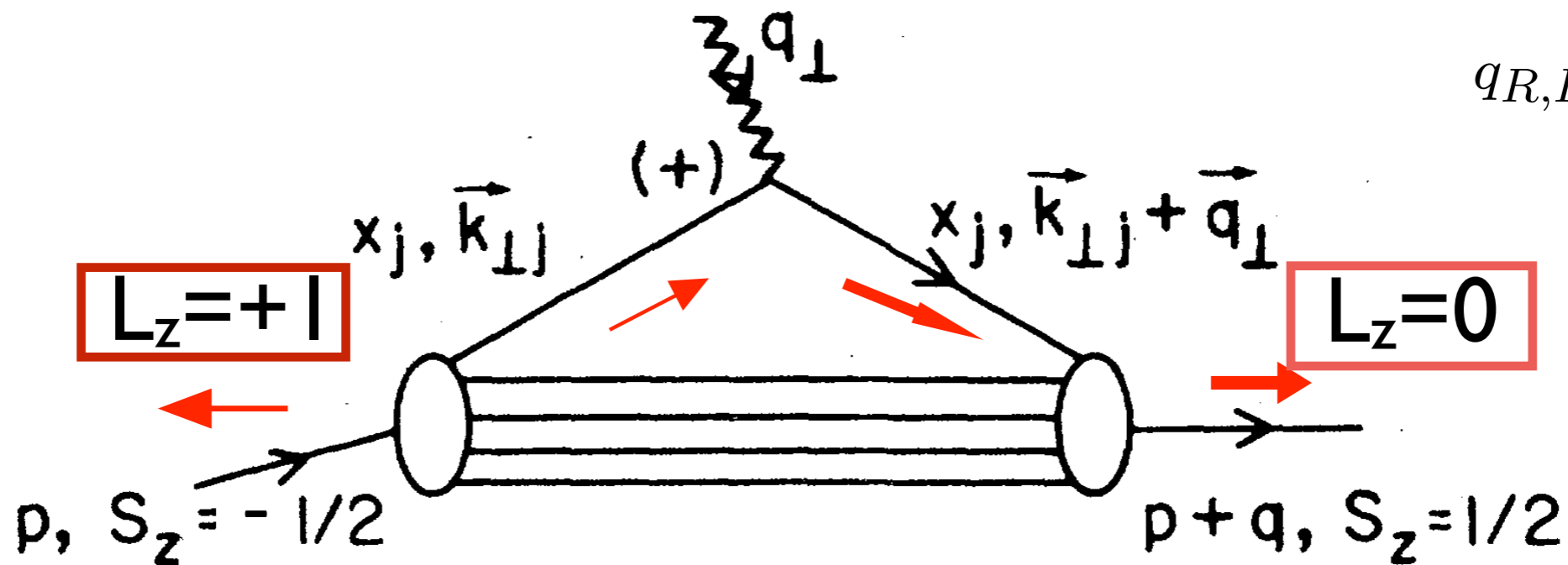
$$\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2\mathbf{k}_\perp] \sum_j e_j \frac{1}{2} \times \quad \text{Drell, sjb}$$

$$\left[ -\frac{1}{q^L} \psi_a^{\uparrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\downarrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\uparrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp$$

$$\mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j) \mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm iq^y$$



Must have  $\Delta l_z = \pm 1$  to have nonzero  $F_2(q^2)$

Nonzero Proton Anomalous Moment  $\rightarrow$   
 Nonzero orbital quark angular momentum

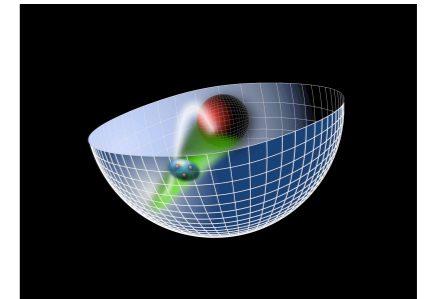
# Underlying Principles

- **Polncarè Invariance: Independent of the observer's Lorentz frame: Quantization at Fixed Light-Front Time  $\tau$**

- **Causality: Information within causal horizon: Light-Front**

- **Light-Front Holography:  $AdS_5 = LF (3+1)$**

$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_{\perp}^2 x(1-x)$$



- **Introduce mass scale  $\kappa$  while retaining the Conformal Invariance of the Action (dAFF)**

*“Emergent Mass”*

- **Unique Dilaton in  $AdS_5$ :  $e^{+\kappa^2 z^2}$**

- **Unique color-confining LF Potential  $U(\zeta^2) = \kappa^4 \zeta^2$**

- **Superconformal Algebra: Mass Degenerate 4-Plet:**

Meson  $q\bar{q} \leftrightarrow$  Baryon  $q[qq] \leftrightarrow$  Tetraquark  $[qq][\bar{q}\bar{q}]$

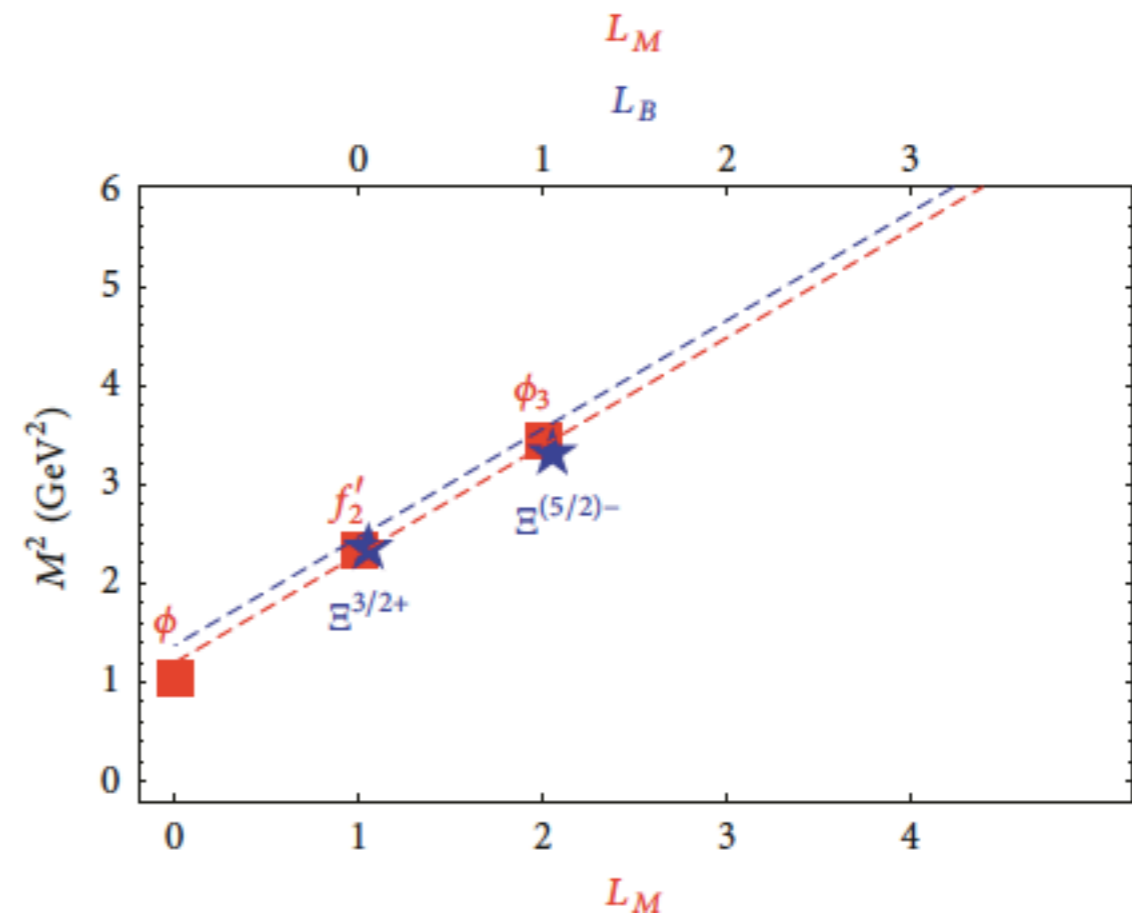
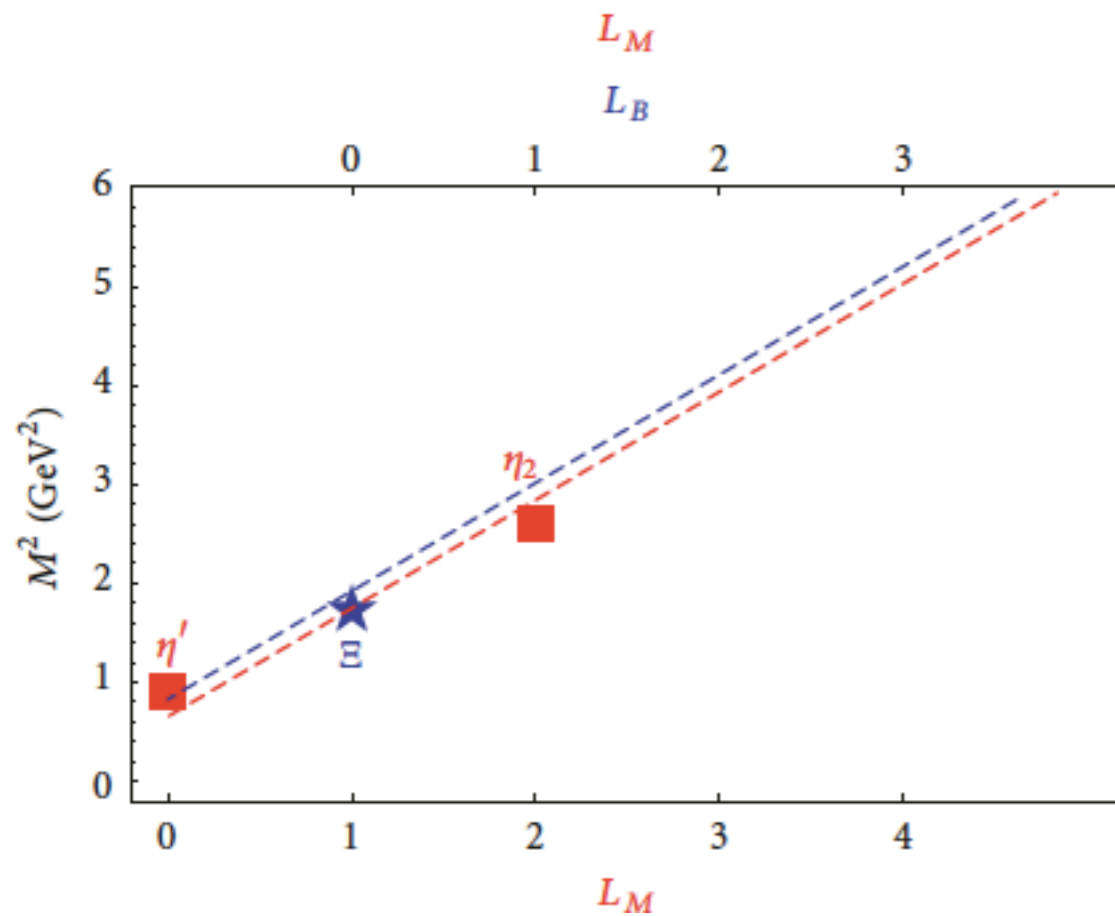
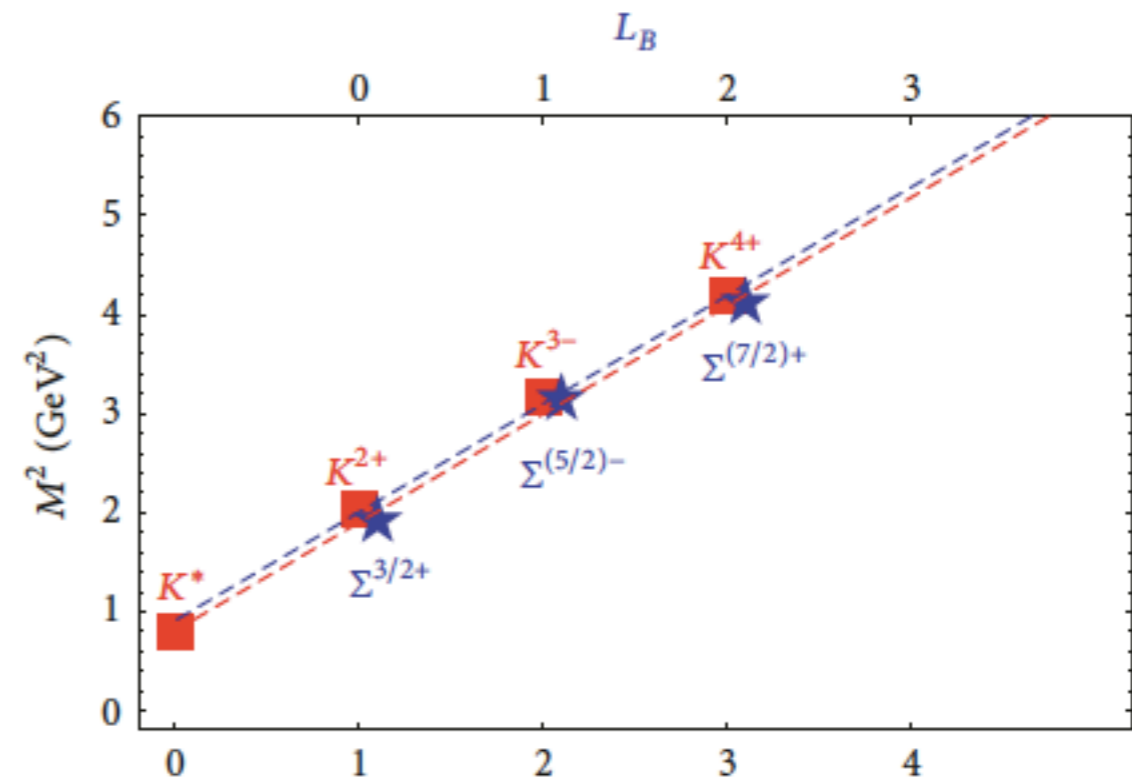
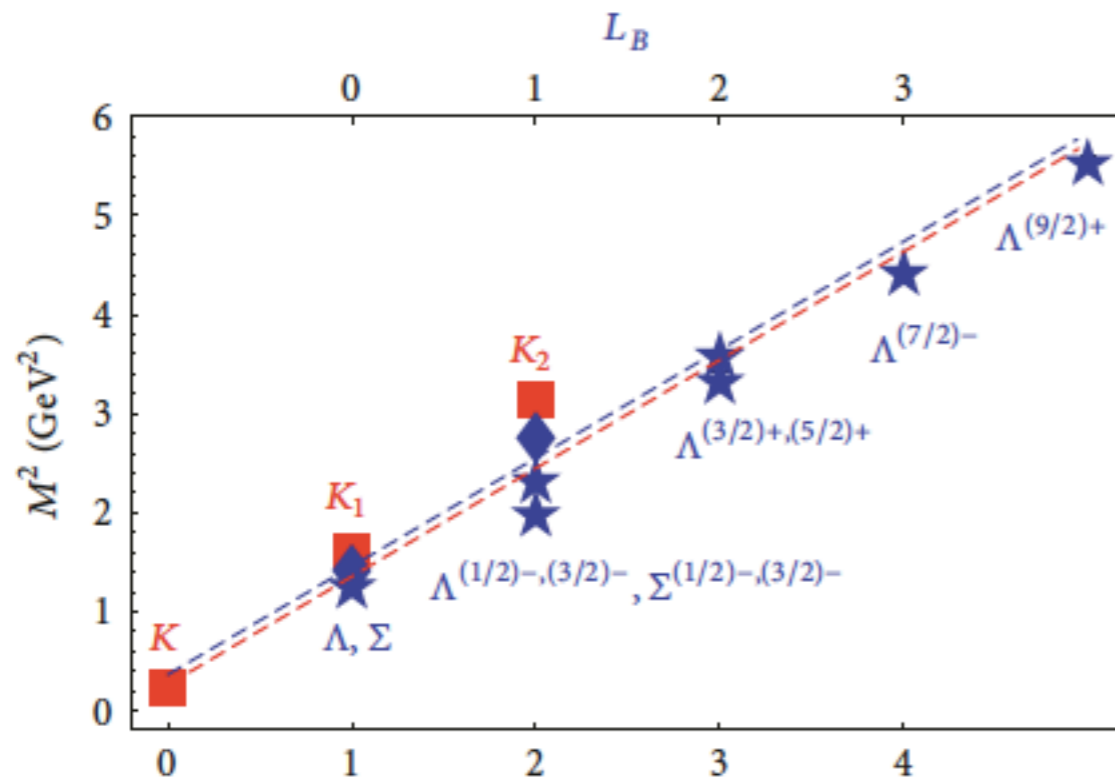
**Stan Brodsky**  
**Bled Workshop**

**Supersymmetric Features of Hadron Physics**  
**from Superconformal Algebra**  
**and Light-Front Holography**

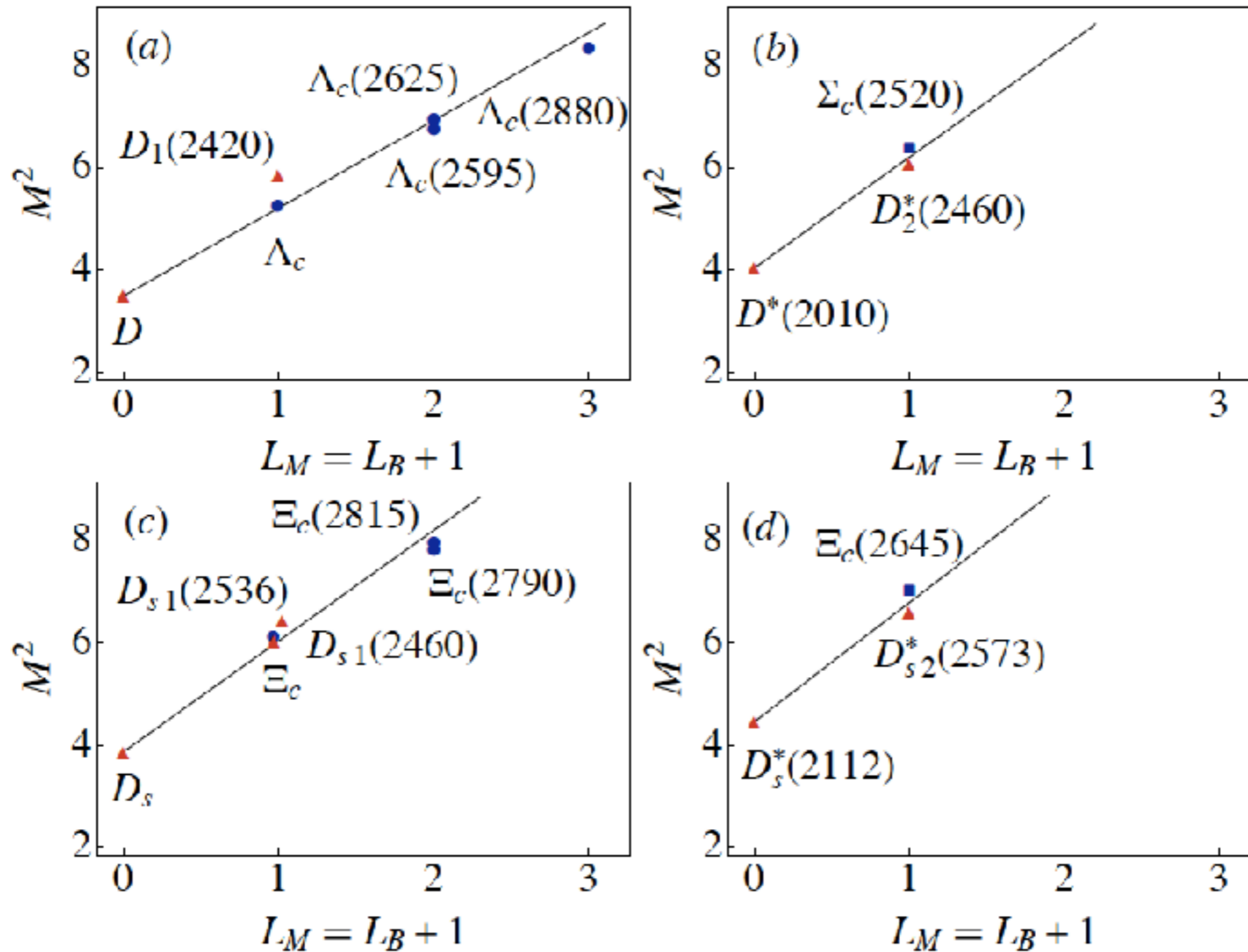


7 July 2021

# Supersymmetry across the light and heavy-light spectrum



# Supersymmetry across the light and heavy-light spectrum



**Heavy charm quark mass does not break supersymmetry**



# Superpartners for states with one c quark

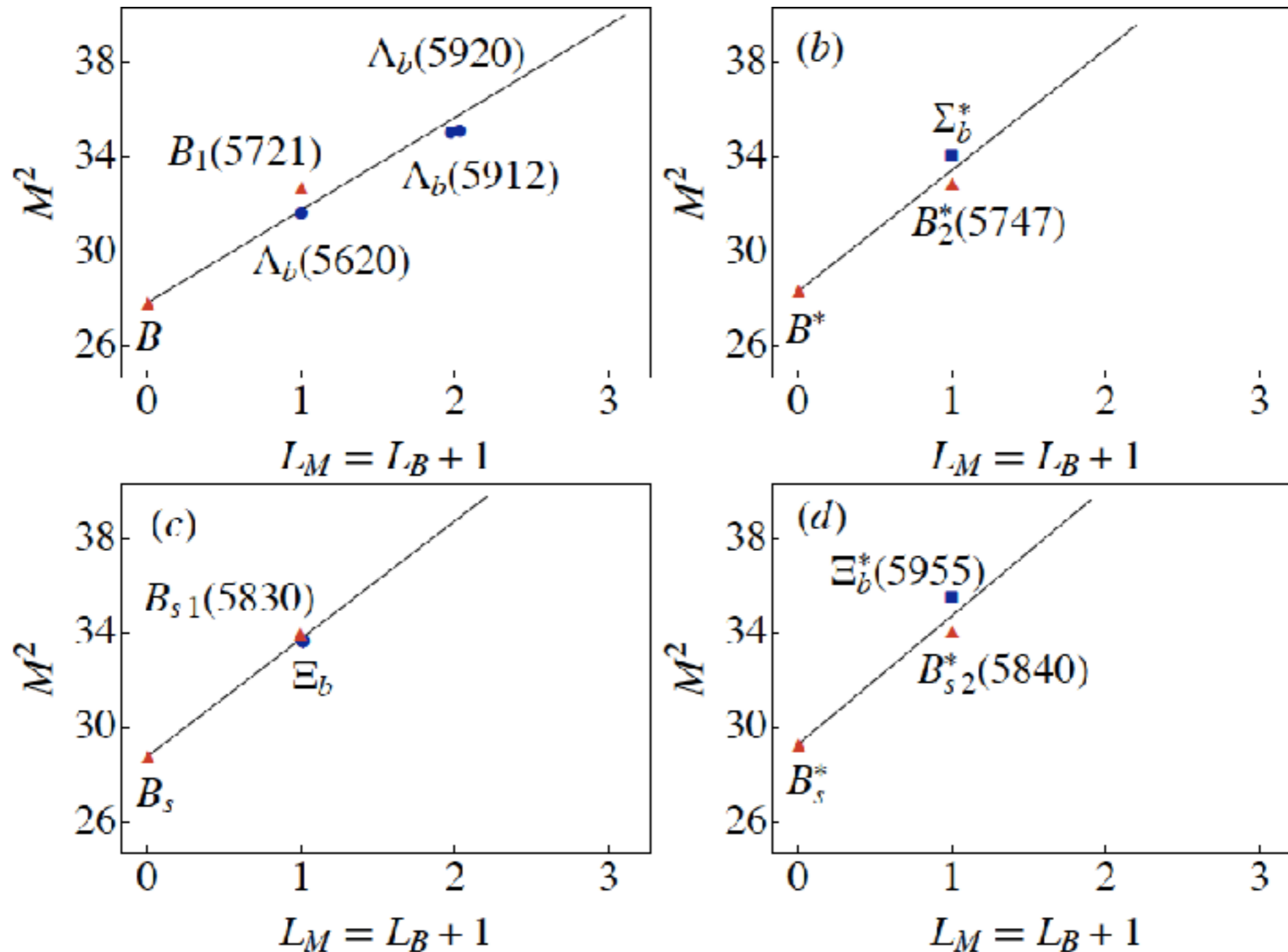
Meson			Baryon			Tetraquark		
$q$ -cont	$J^{P(C)}$	Name	$q$ -cont	$J^P$	Name	$q$ -cont	$J^{P(C)}$	Name
$\bar{q}c$	$0^-$	$D(1870)$	—	—	—	—	—	—
$\bar{q}c$	$1^+$	$D_1(2420)$	$[ud]c$	$(1/2)^+$	$\Lambda_c(2290)$	$[ud][\bar{c}\bar{q}]$	$0^+$	$\bar{D}_0^*(2400)$
$\bar{q}c$	$2^-$	$D_J(2600)$	$[ud]c$	$(3/2)^-$	$\Lambda_c(2625)$	$[ud][\bar{c}\bar{q}]$	$1^-$	—
$\bar{c}q$	$0^-$	$\bar{D}(1870)$	—	—	—	—	—	—
$\bar{c}q$	$1^+$	$\bar{D}_1(2420)$	$[cq]q$	$(1/2)^+$	$\Sigma_c(2455)$	$[cq][\bar{u}\bar{d}]$	$0^+$	$D_0^*(2400)$
$\bar{q}c$	$1^-$	$D^*(2010)$	—	—	—	—	—	—
$\bar{q}c$	$2^+$	$D_2^*(2460)$	$(qq)c$	$(3/2)^+$	$\Sigma_c^*(2520)$	$(qq)[\bar{c}\bar{q}]$	$1^+$	$D(2550)$
$\bar{q}c$	$3^-$	$D_3^*(2750)$	$(qq)c$	$(3/2)^-$	$\Sigma_c(2800)$	$(qq)[\bar{c}\bar{q}]$	—	—
$\bar{s}c$	$0^-$	$D_s(1968)$	—	—	—	—	—	—
$\bar{s}c$	$1^+$	$D_{s1}(2460)$	$[qs]c$	$(1/2)^+$	$\Xi_c(2470)$	$[qs][\bar{c}\bar{q}]$	$0^+$	$\bar{D}_{s0}^*(2317)$
$\bar{s}c$	$2^-$	$D_{s2}(\sim 2860)?$	$[qs]c$	$(3/2)^-$	$\Xi_c(2815)$	$[sq][\bar{c}\bar{q}]$	$1^-$	—
$\bar{s}c$	$1^-$	$D_s^*(2110)$	—	—	—	—	—	—
$\bar{s}c$	$2^+$	$D_{s2}^*(2573)$	$(sq)c$	$(3/2)^+$	$\Xi_c^*(2645)$	$(sq)[\bar{c}\bar{q}]$	$1^+$	$D_{s1}(2536)$
$\bar{c}s$	$1^+$	$\bar{D}_{s1}(\sim 2700)?$	$[cs]s$	$(1/2)^+$	$\Omega_c(2695)$	$[cs][\bar{s}\bar{q}]$	$0^+$	??
$\bar{s}c$	$2^+$	$D_{s2}^*(\sim 2750)?$	$(ss)c$	$(3/2)^+$	$\Omega_c(2770)$	$(ss)[\bar{c}\bar{s}]$	$1^+$	??

M. Nielsen, sjb

predictions

beautiful agreement!

# Supersymmetry across the light and heavy-light spectrum



Heavy bottom quark mass does not break supersymmetry

# Heavy-light and heavy-heavy hadronic sectors

- Extension to the heavy-light hadronic sector

[H. G. Dosch, GdT, S. J. Brodsky, PRD **92**, 074010 (2015), PRD **95**, 034016 (2017)]

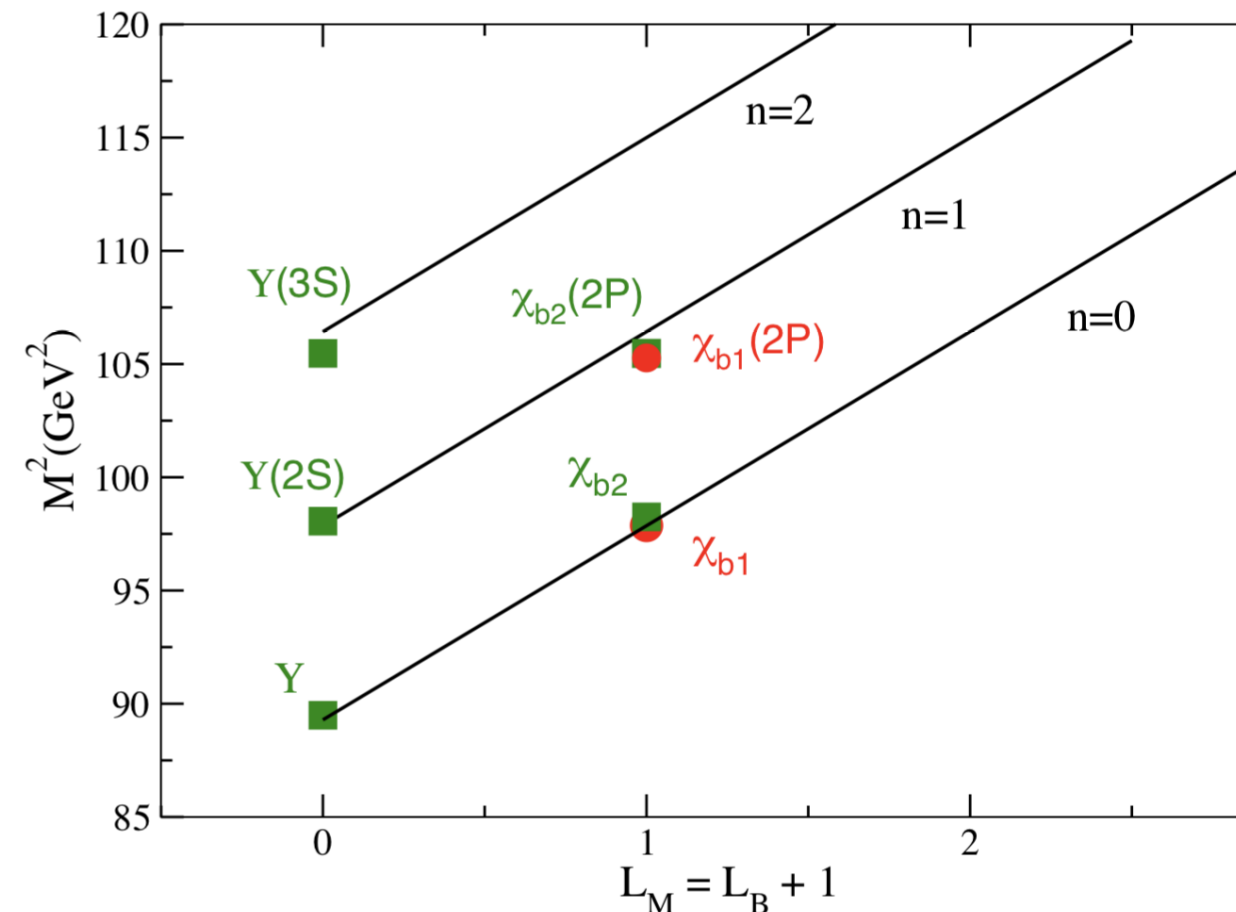
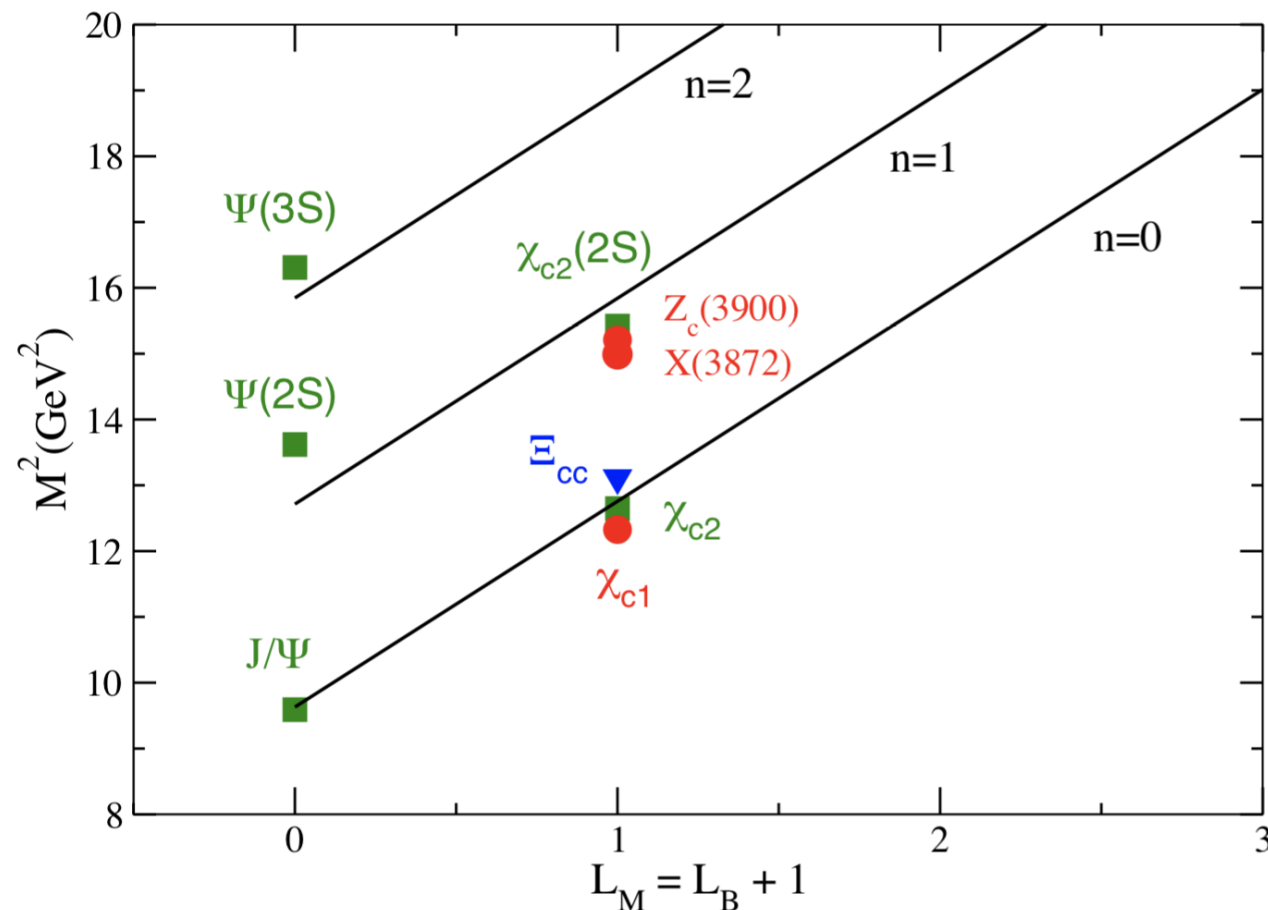
- Extension to the double-heavy hadronic sector

[M. Nielsen and S. J. Brodsky, PRD, 114001 (2018)]

[M. Nielsen, S. J. Brodsky, GdT, H. G. Dosch, F. S. Navarra, L. Zou, PRD **98**, 034002 (2018)]

- Extension to the isoscalar hadronic sector

[L. Zou, H. G. Dosch, GdT, S. J. Brodsky, arXiv:1901.11205 [hep-ph]]



# Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS<sub>5</sub> space in dilaton background  $\varphi(z) = \kappa^2 z^2$

$$e^{\phi(z)} = e^{+\kappa^2 z^2} \quad S = -\frac{1}{4} \int d^4x dz \sqrt{g} e^{\varphi(z)} \frac{1}{g_5^2} G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling  $g_5(z)$  incorporates the non-conformal dynamics of confinement

- YM coupling  $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$  is the five dim coupling up to a factor:  $g_5(z) \rightarrow g_{YM}(\zeta)$
- Coupling measured at momentum scale  $Q$

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s^{AdS}(\zeta)$$

- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) e^{-Q^2/4\kappa^2}$$

where the coupling  $\alpha_s^{AdS}$  incorporates the non-conformal dynamics of confinement

Bjorken sum rule defines effective charge

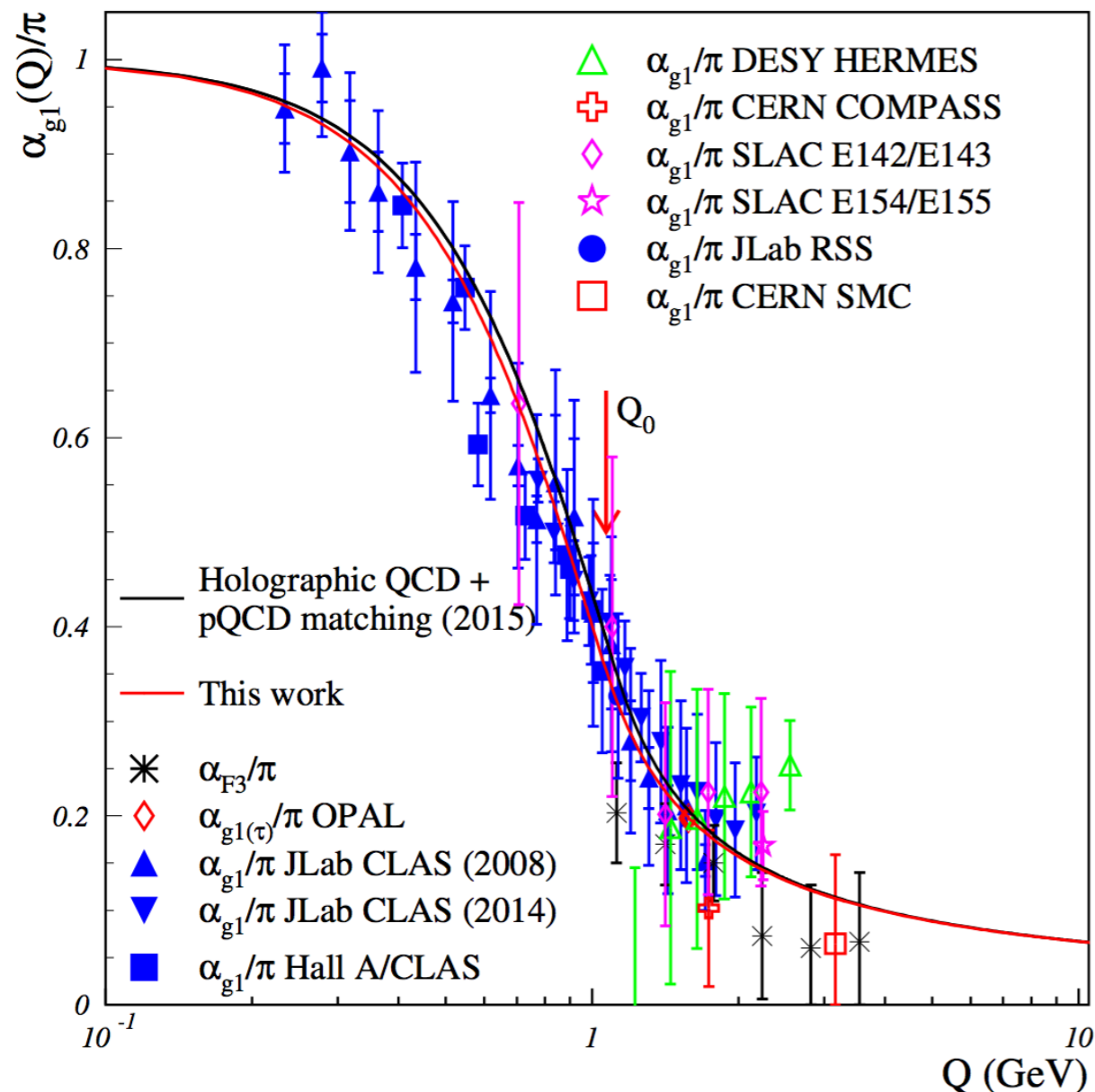
$$\alpha_{g1}(Q^2)$$

$$\int_0^1 dx [g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2)] \equiv \frac{g_a}{6} \left[ 1 - \frac{\alpha_{g1}(Q^2)}{\pi} \right]$$

- **Can be used as standard QCD coupling**
- **Well measured**
- **Asymptotic freedom at large  $Q^2$**
- **Computable at large  $Q^2$  in any pQCD scheme**
- **Universal  $\beta_0, \beta_1$**



# Running Coupling from AdS/QCD



Bjorken sum rule:

$$\frac{\alpha_{g_1}(Q^2)}{\pi} = 1 - \frac{6}{g_A} \int_0^1 dx g_1^{p-n}(x, Q^2)$$

Effective coupling in LFHQCD  
(valid at low- $Q^2$ )

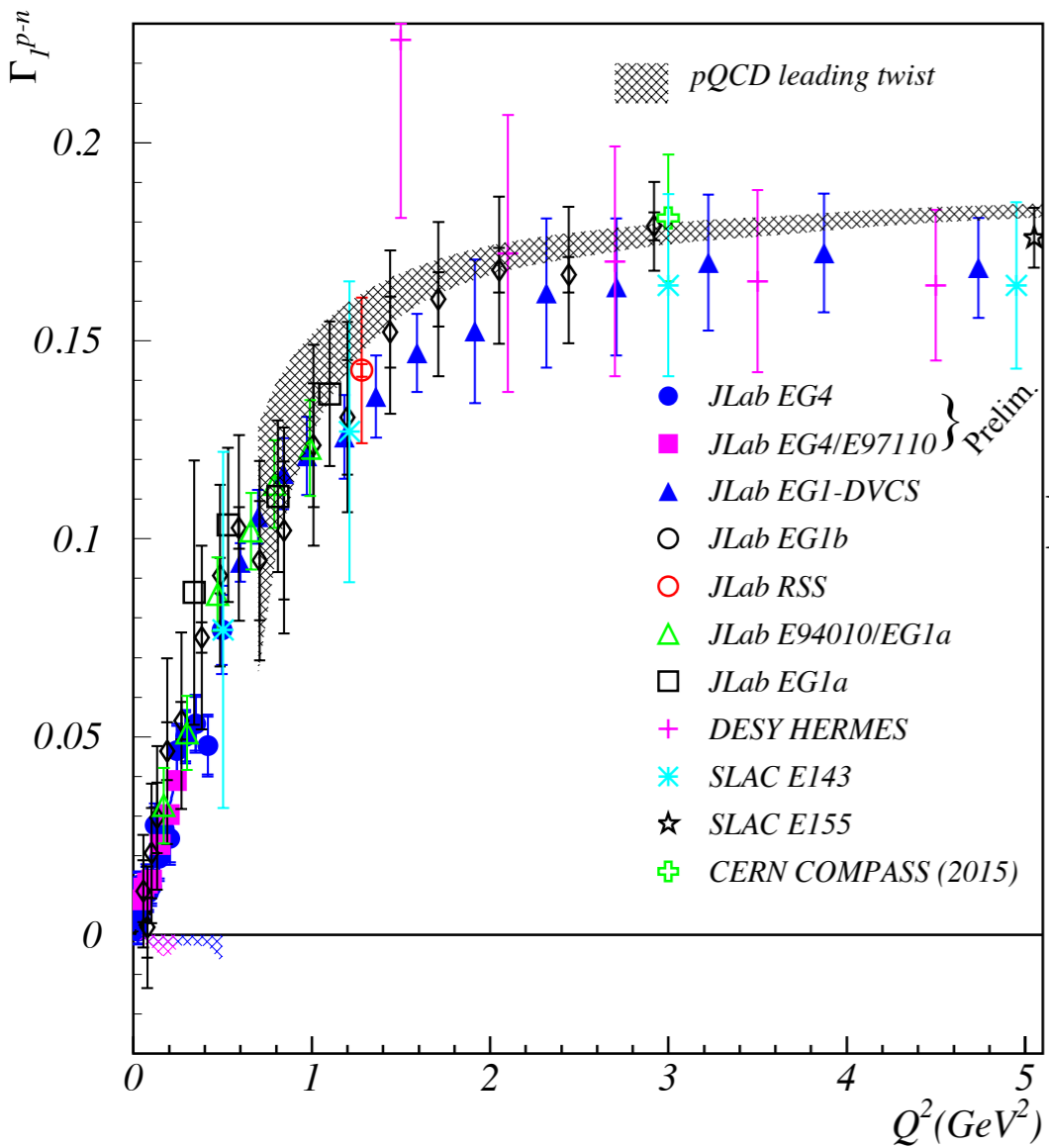
$$\alpha_{g_1}^{AdS}(Q^2) = \pi \exp(-Q^2/4\kappa^2)$$

Imposing continuity for  $\alpha$   
and its first derivative

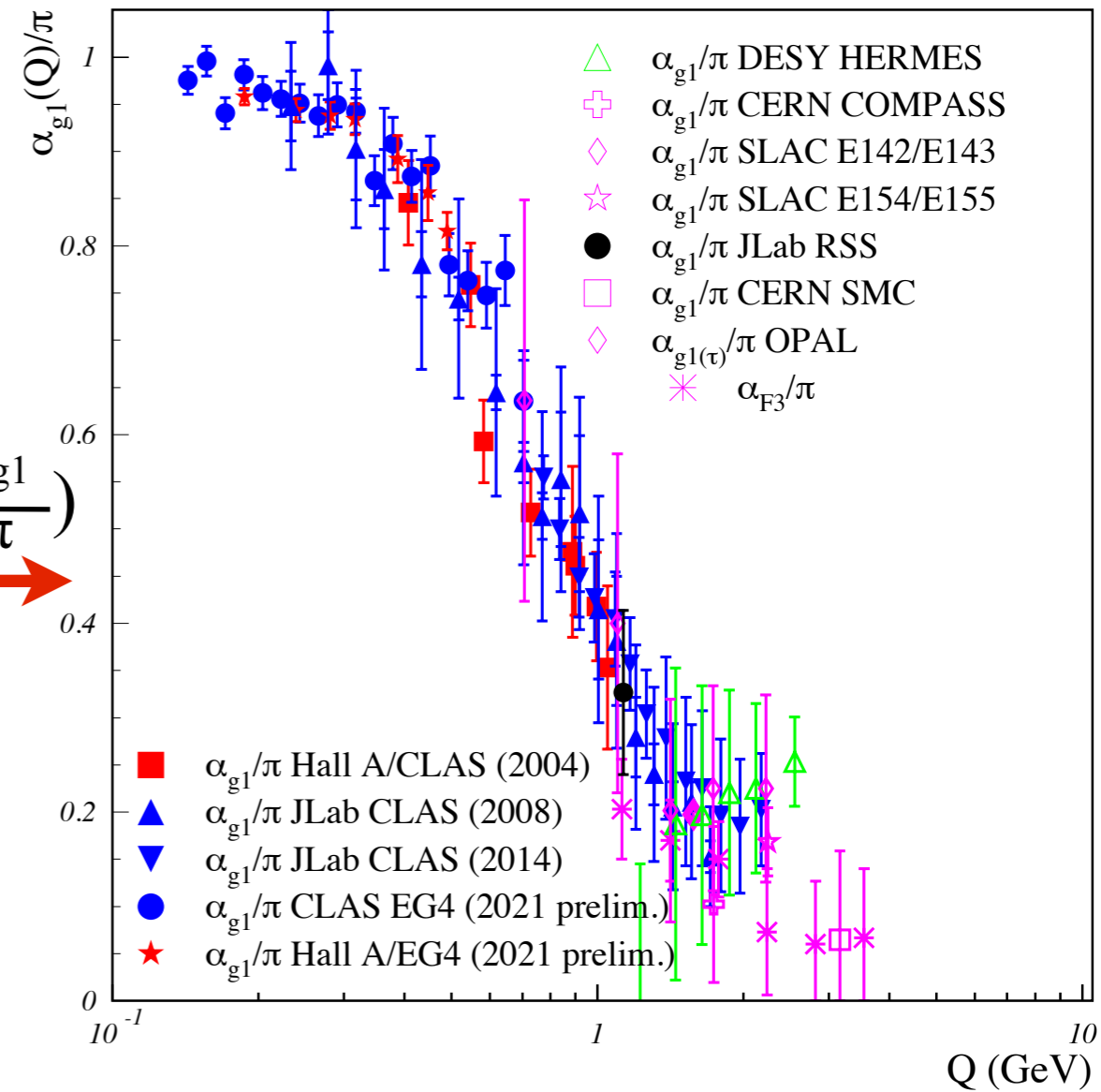
A. Deur, S.J. Brodsky, G.F. de Téramond,  
Phys. Lett. B 750, 528 (2015); J. Phys. G 44, 105005 (2017).

**Analytic, defined at all scales, IR Fixed Point**

# Bjorken sum $\Gamma_1^{p-n}$ measurements



$$\Gamma_1^{p-n} \triangleq \frac{1}{6} g_A \left( 1 - \frac{\alpha_{g1}}{\pi} \right)$$



# Low $Q^2$ limit

A. Deur

At  $Q^2 = 0$ , a sum rule related to the Bjorken sum rule exists: the Gerasimov-Drell-Hearn (GDH) sum rule:

At  $Q^2 = 0$ , GDH sum rule:

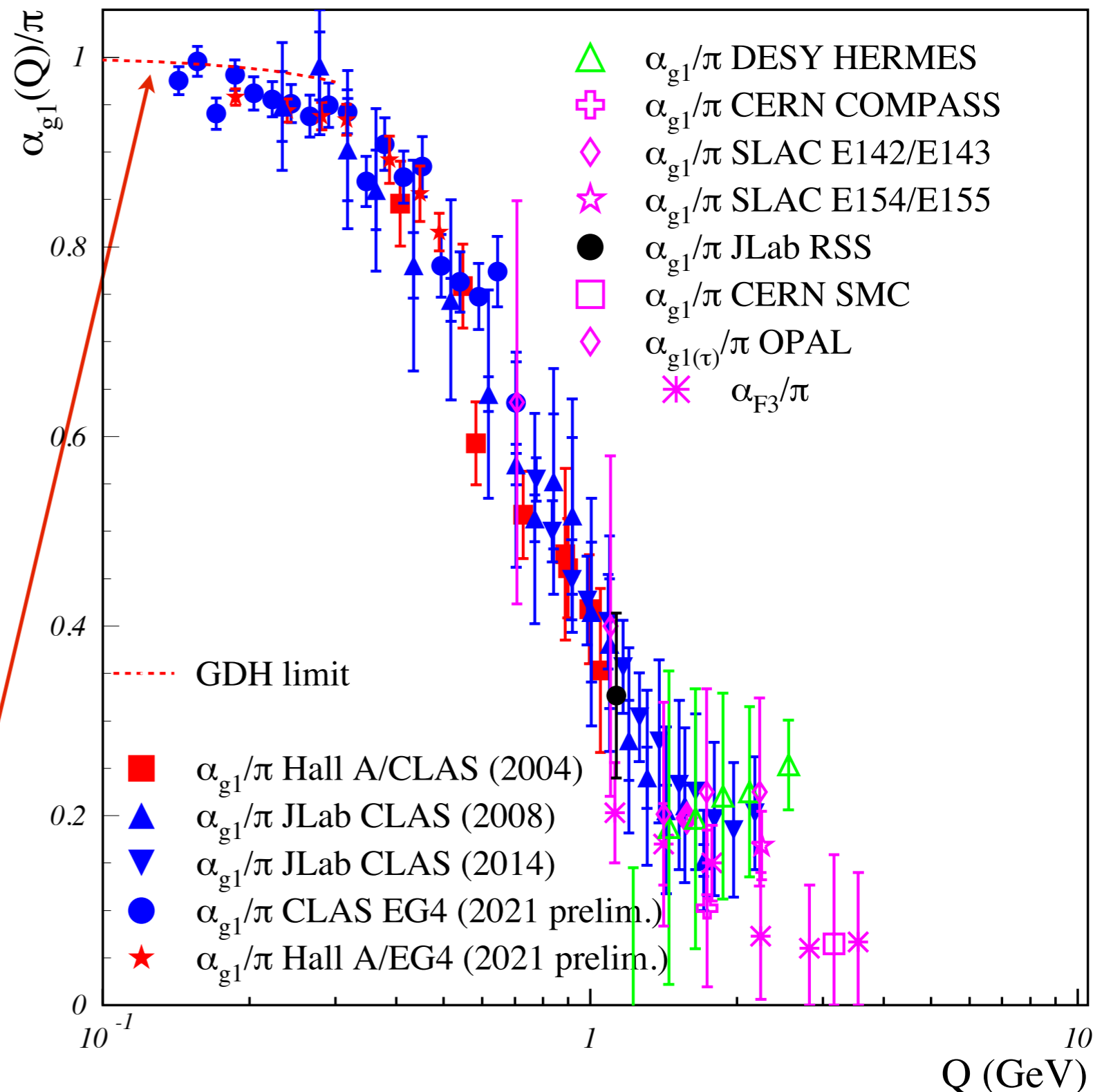
$$\Gamma_1 = \frac{-\kappa^2 Q^2}{8M^2}$$

↙ anomalous magnetic moment      ↘ Nucleon mass

$\Rightarrow Q^2 = 0$  constraints:

$$\Rightarrow \begin{cases} \alpha_{g1} = \pi \\ \frac{d\alpha_{g1}}{dQ^2} = \frac{3\pi}{4g_A} \left( \frac{\kappa_n^2}{M_n^2} - \frac{\kappa_p^2}{M_p^2} \right) \end{cases}$$

↖ 3.66      ↗ 3.20



First experimental evidence of nearly *conformal behavior* (i.e. no  $Q^2$ -dependence) of QCD at low  $Q^2$ .

$$m_\rho = \sqrt{2}\kappa$$

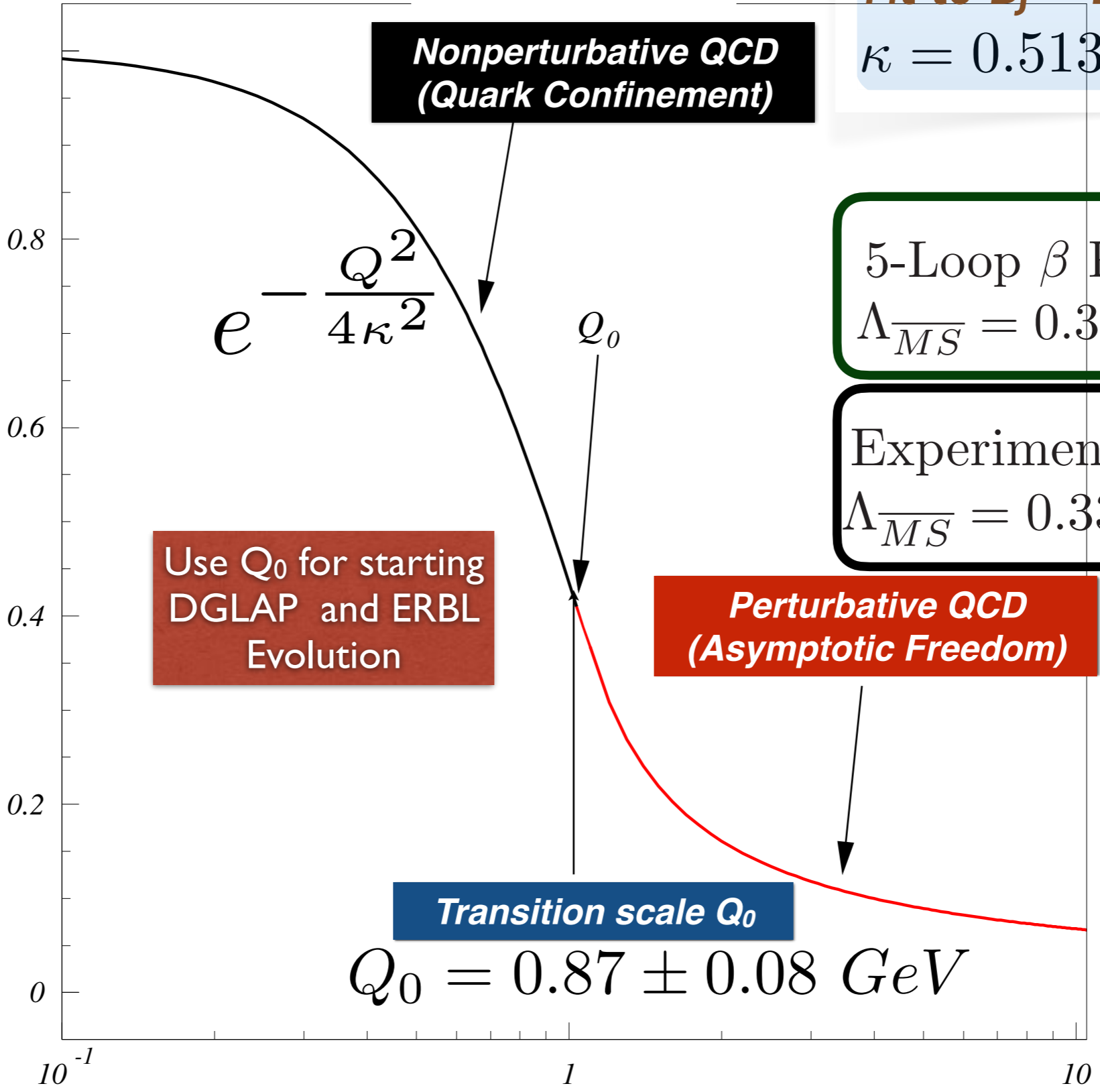
$$m_p = 2\kappa$$

Deur, de Tèramond, sjb

**All-Scale QCD Coupling**

Fit to Bj + DHG Sum Rules:  
 $\kappa = 0.513 \pm 0.007 \text{ GeV}$

$$\frac{\alpha_{g_1}^s(Q^2)}{\pi}$$



5-Loop  $\beta$  Prediction:  
 $\Lambda_{\overline{MS}} = 0.339 \pm 0.019 \text{ GeV}$

Experiment:  
 $\Lambda_{\overline{MS}} = 0.332 \pm 0.017 \text{ GeV}$

Use  $Q_0$  for starting  
 DGLAP and ERBL  
 Evolution

**Perturbative QCD  
 (Asymptotic Freedom)**

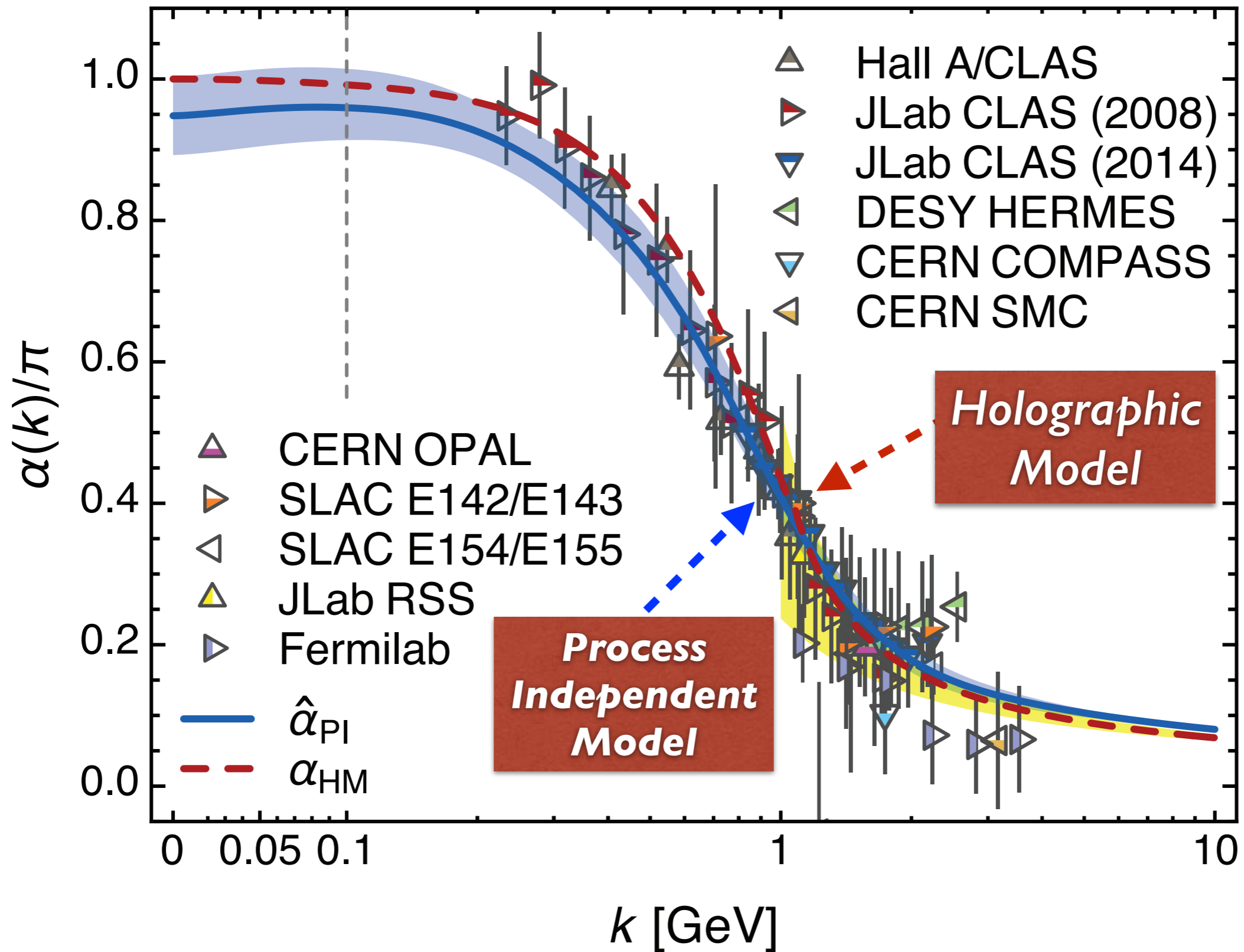
**Transition scale  $Q_0$**

$$Q_0 = 0.87 \pm 0.08 \text{ GeV}$$

$$\lambda \equiv \kappa^2$$

*Reverse Dimensional Transmutation!*

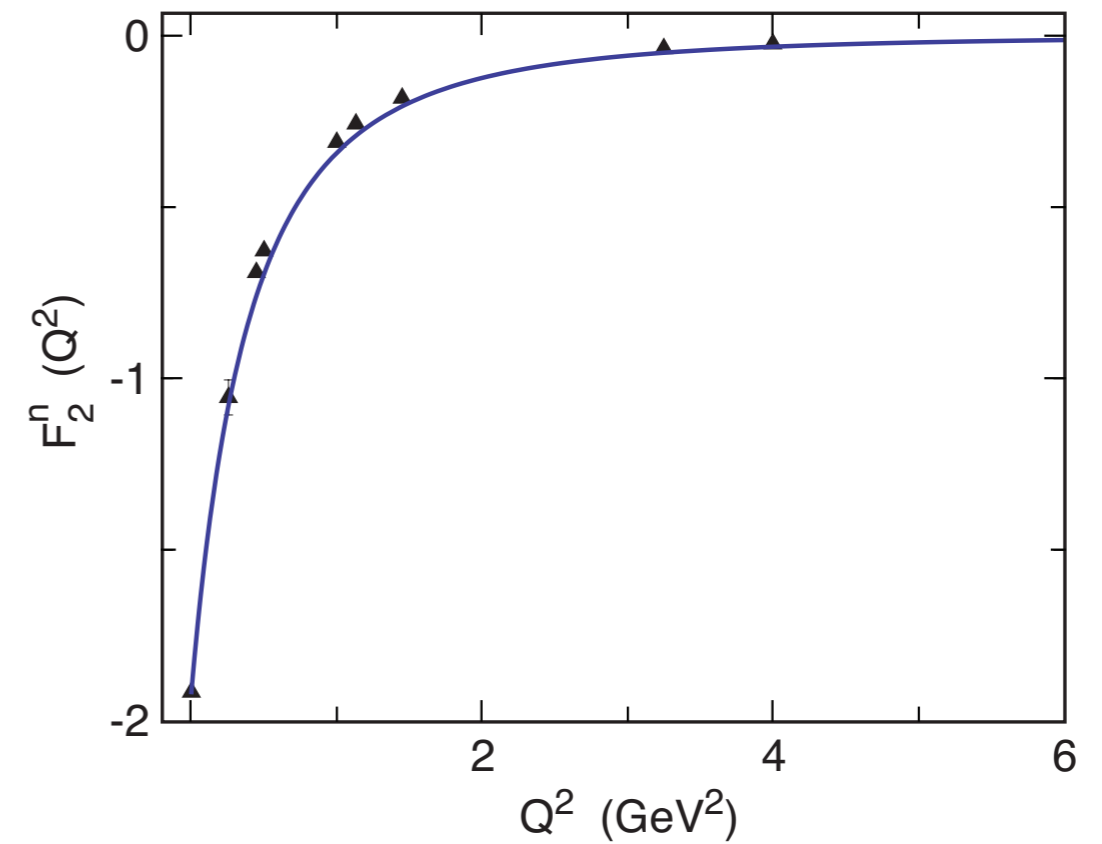
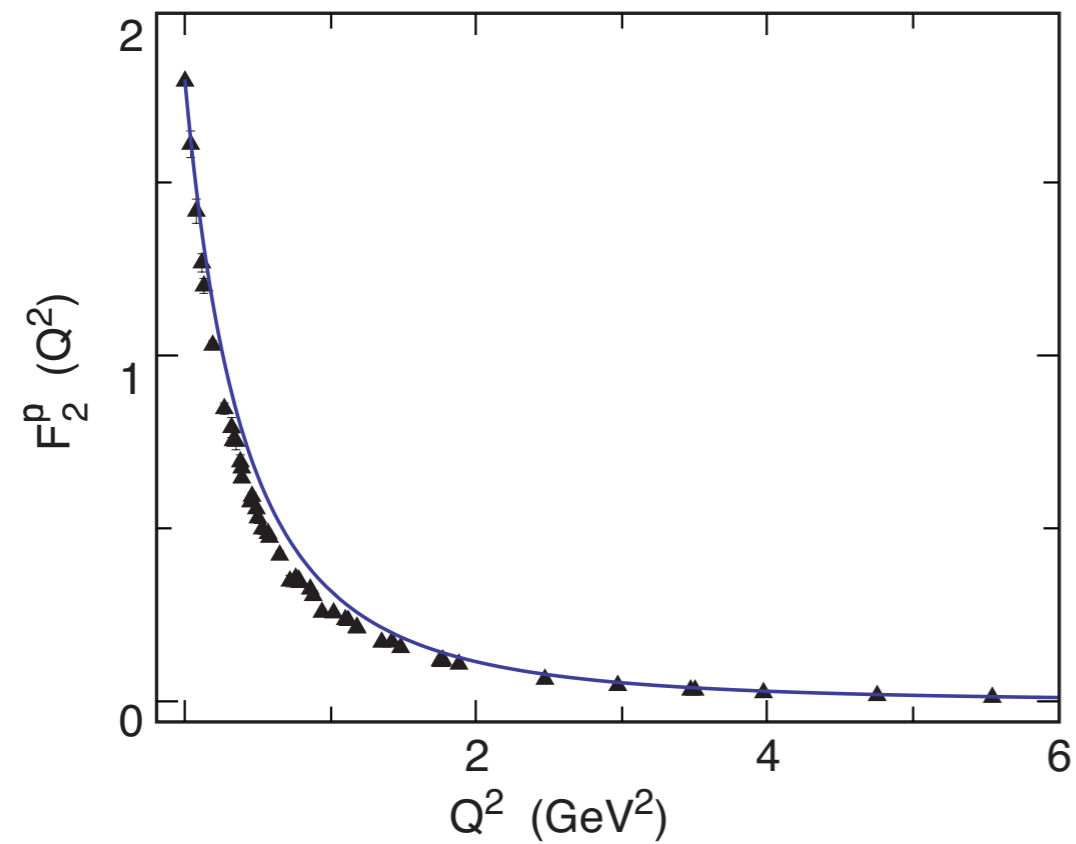
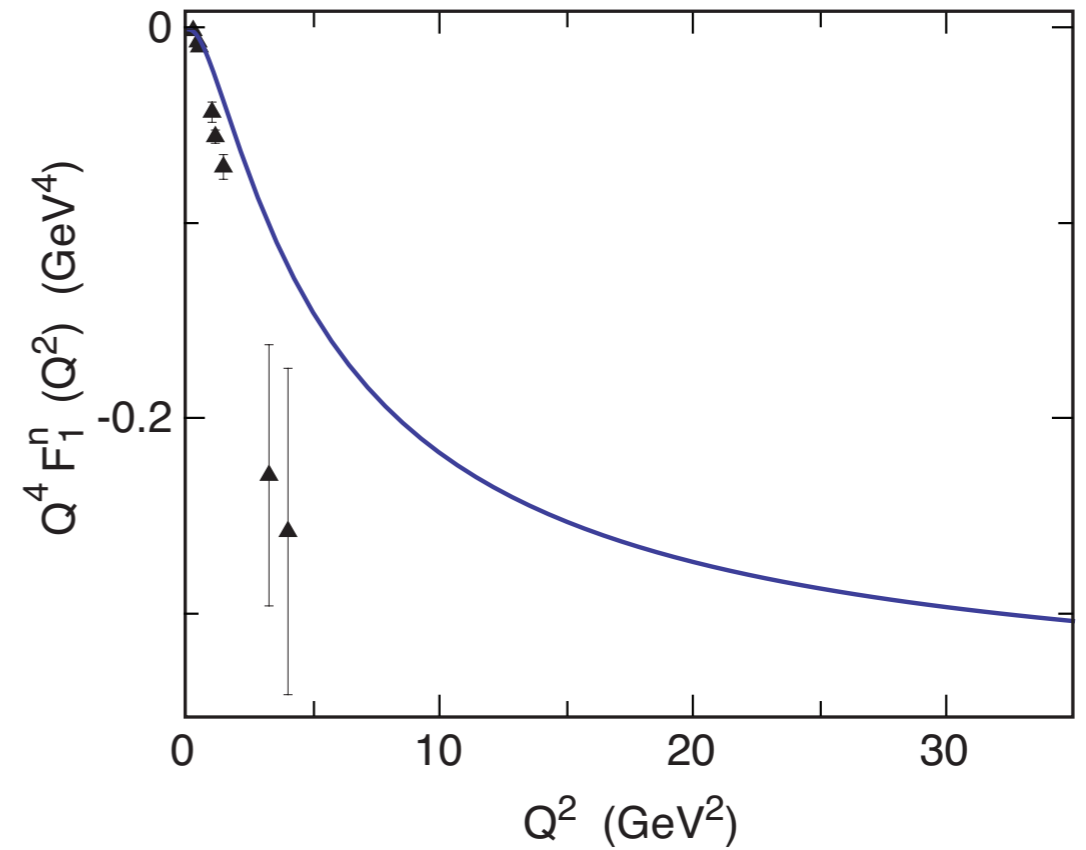
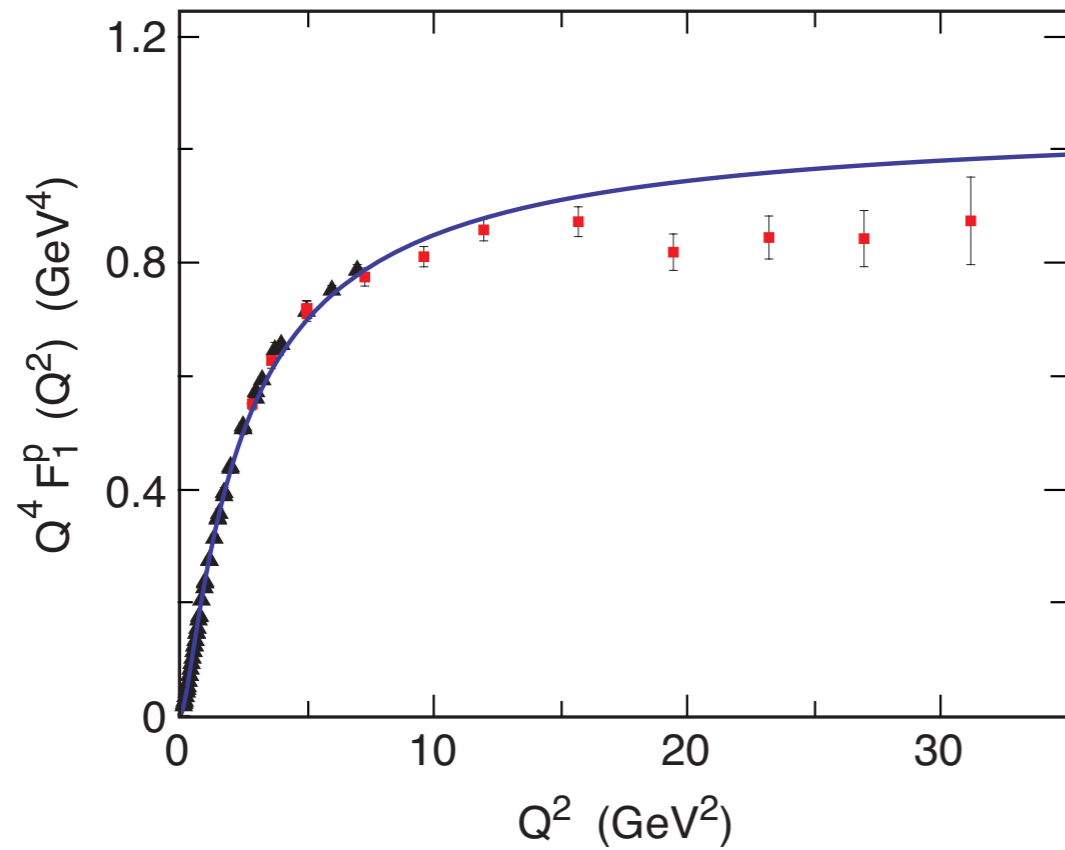
$\overline{MS}$  scheme



Process-independent strong running coupling

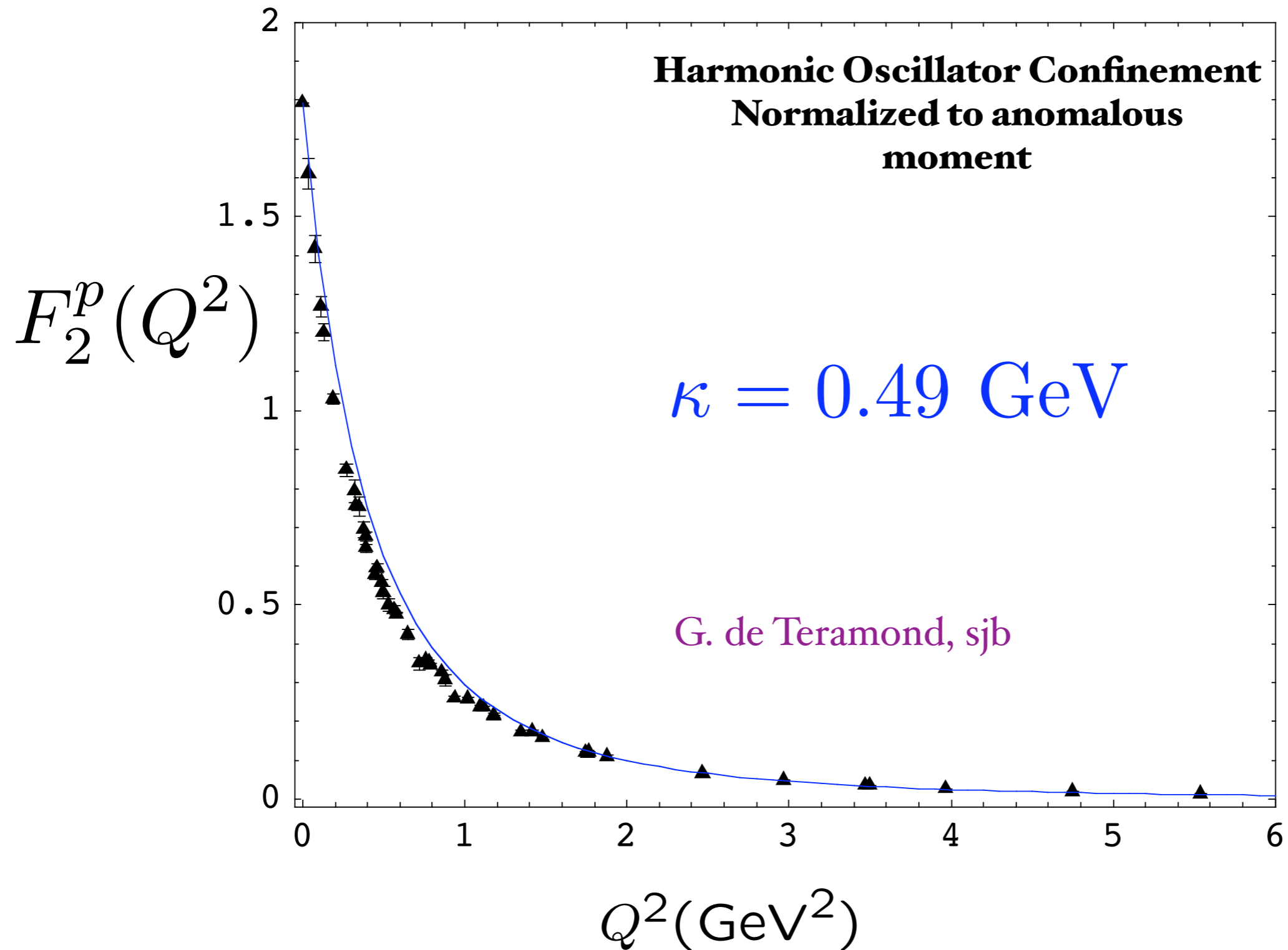


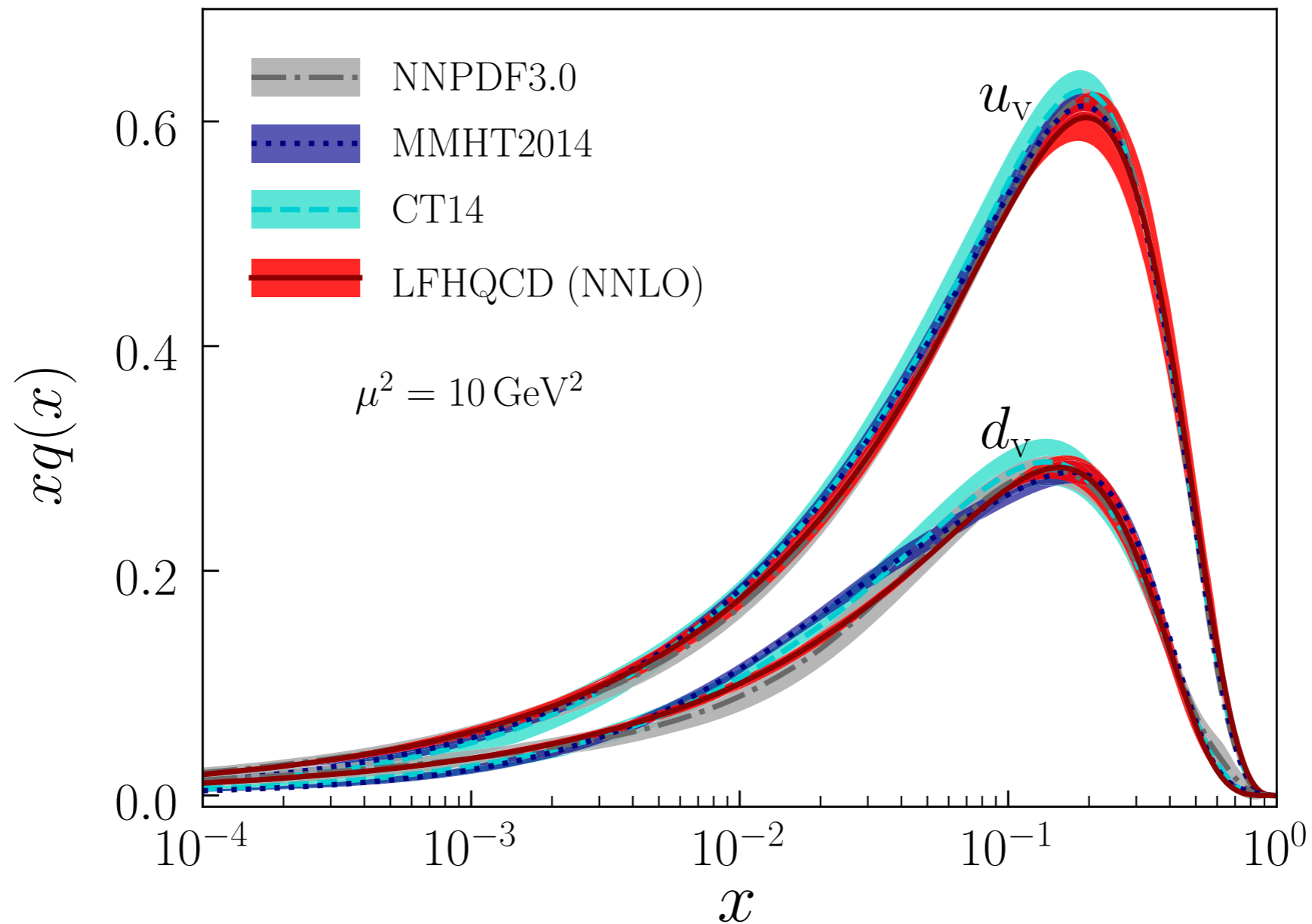
Using  $SU(6)$  flavor symmetry and normalization to static quantities



# Spacelike Pauli Form Factor

From overlap of  $L = 1$  and  $L = 0$  LFWFs





Comparison for  $xq(x)$  in the proton from LFHQCD (red bands) and global fits: MMHT2014 (blue bands) [5], CT14 [6] (cyan bands), and NNPDF3.0 (gray bands) [77]. LFHQCD results are evolved from the initial scale  $\mu_0 = 1.06 \pm 0.15$  GeV.

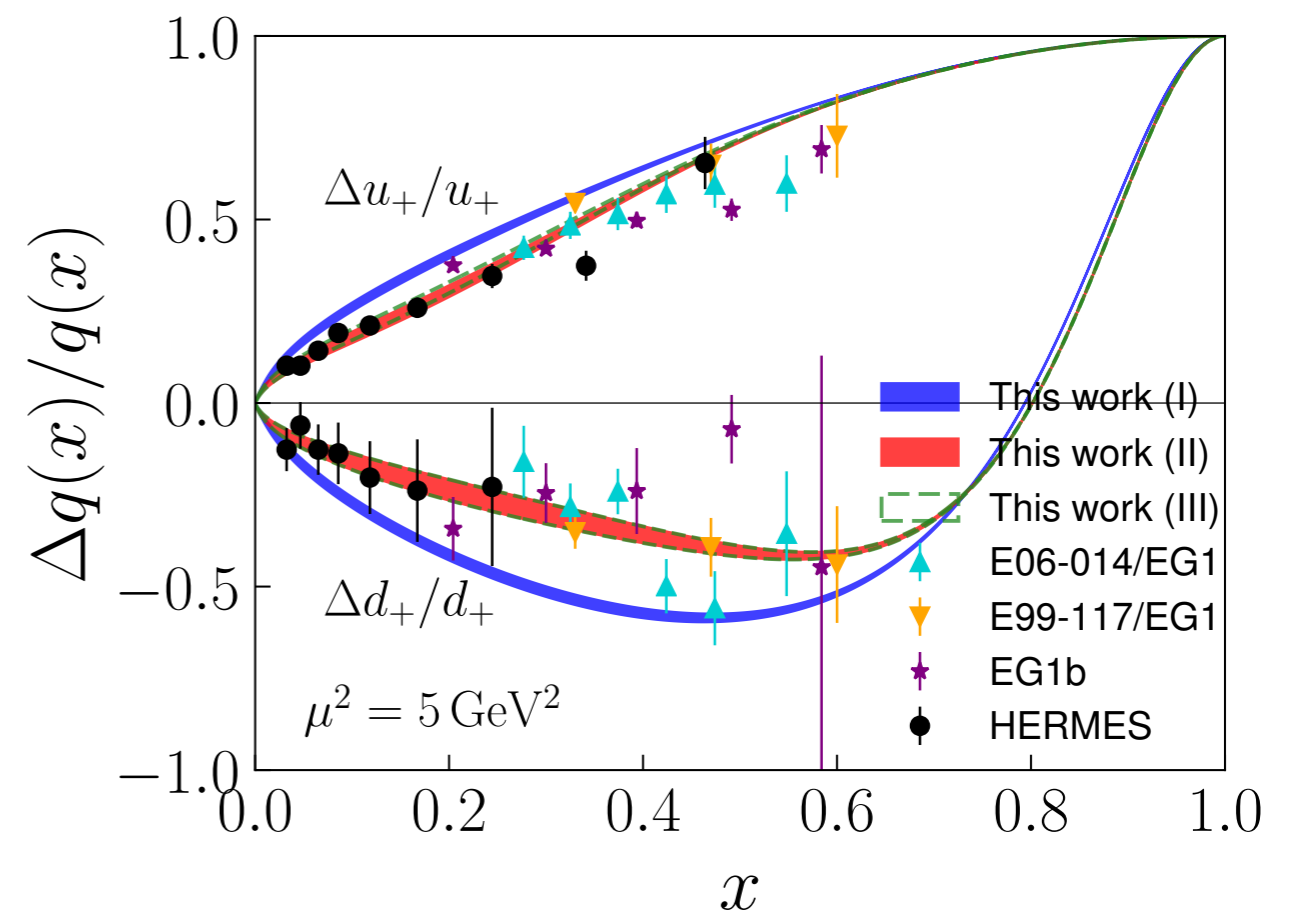
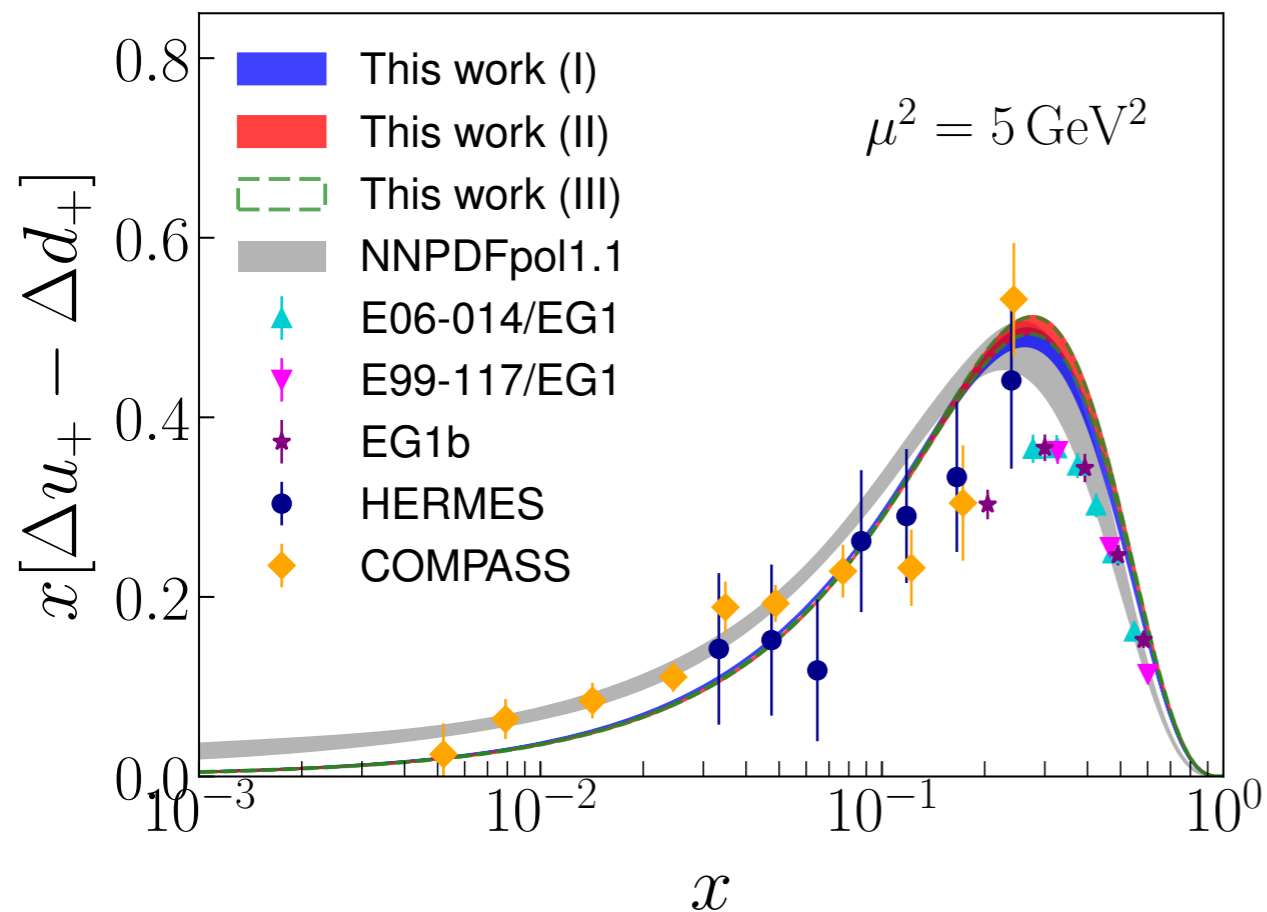
*Universality of Generalized Parton Distributions in Light-Front Holographic QCD*

*Guy F. de Téramond, Tianbo Liu, Raza Sabbir Sufian, Hans Günter Dosch, Stanley J. Brodsky, and Alexandre Deur*

*PHYSICAL REVIEW LETTERS 120, 182001 (2018)*

## Polarized GPDs and PDFs (HLFHS Collaboration, 2019)

- Separation of chiralities in the AdS action allows computation of the matrix elements of the axial current including the correct normalization, once the coefficients  $c_T$  are fixed for the vector current
- Helicity retention between quark and parent hadron (pQCD prediction):  $\lim_{x \rightarrow 1} \frac{\Delta q(x)}{q(x)} = 1$
- No spin correlation with parent hadron:  $\lim_{x \rightarrow 0} \frac{\Delta q(x)}{q(x)} = 0$



Other Consequences of  $[ud]_{\bar{3}_C, I=0, J=0}$  diquark cluster

## QCD Hidden-Color Hexadiquark in the Core of Nuclei

J. Rittenhouse West, G. de Teramond, A. S. Goldhaber, I. Schmidt, sjb

$$|\Psi_{HDQ}\rangle = |[ud][ud][ud][ud][ud][ud]\rangle$$

mixes with

$${}^4He|npnp\rangle$$

Increases alpha binding energy, EMC effects

## Diquarks Can Dominate Five-Quark Fock State of Proton

$$|p\rangle = \alpha|[ud]u\rangle + \beta|[ud][ud]\bar{d}\rangle$$

J. Rittenhouse West, sjb (to be published)

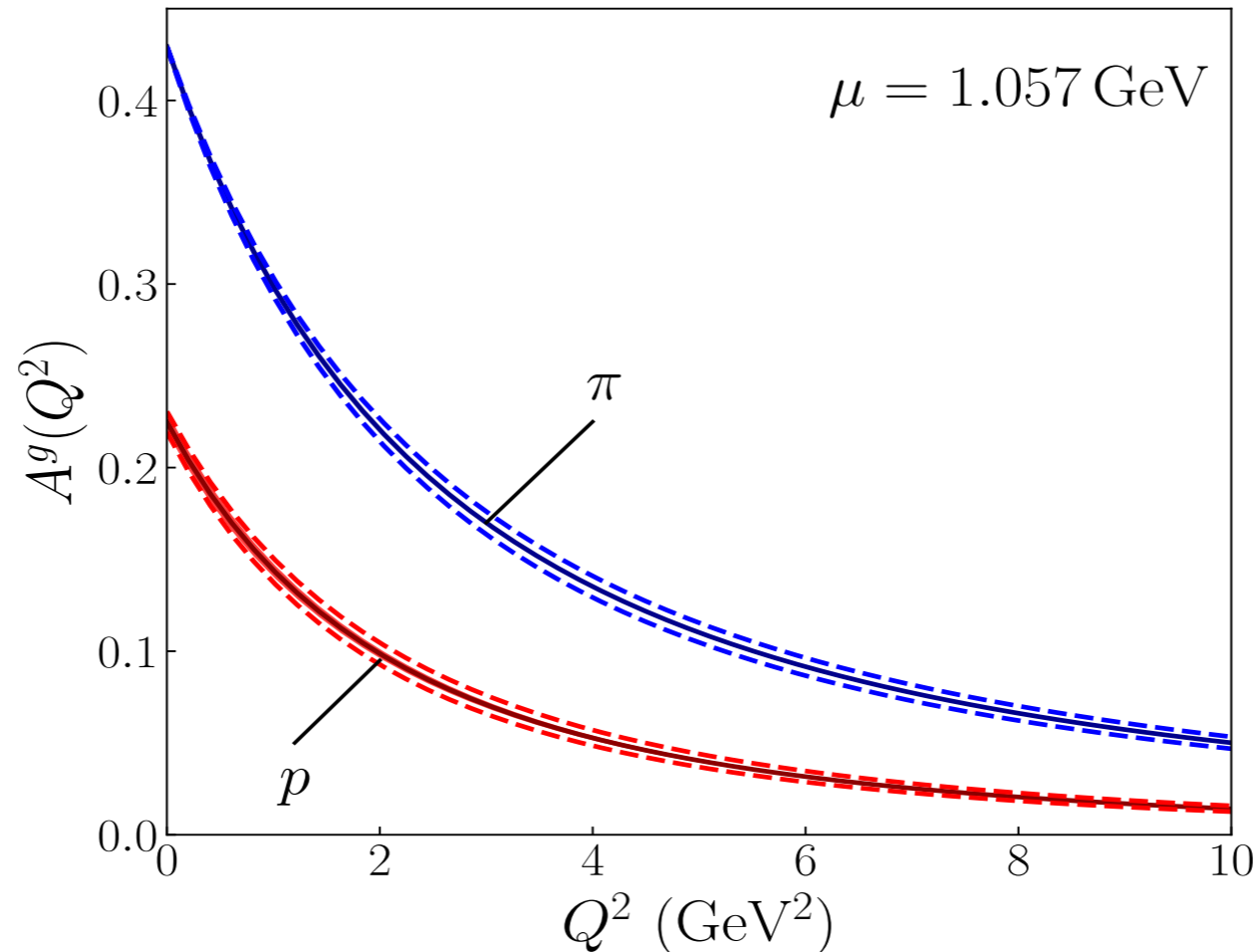
Natural explanation why  $\bar{d}(x) \gg \bar{u}(x)$  in proton



# Gluon matter distribution in the proton and pion from extended holographic light-front QCD

Guy F. de Téramond,<sup>1</sup> H. G. Dosch,<sup>2</sup> Tianbo Liu,<sup>3,\*</sup>  
 Raza Sabbir Sufian,<sup>4,5,†</sup> Stanley J. Brodsky,<sup>6</sup> and Alexandre Deur<sup>5</sup>  
 (HLFHS Collaboration)

$$\langle P' | T_\mu^\nu | P \rangle = (P^\nu P'_\mu + P_\mu P'^\nu) A(Q^2)$$



Gluon gravitational form factor  $A^g(Q^2)$  of the proton (red) and the pion (blue). The dashed curves indicate the uncertainty from the variation of  $\lambda_g$  by  $\pm 5\%$ . The value  $A^g(0)$  corresponds to the momentum fraction carried by gluons: 0.225 for the proton and 0.429 for the pion.

# Gluon matter distribution in the proton and pion from extended holographic light-front QCD

Guy F. de Téramond,<sup>1</sup> H. G. Dosch,<sup>2</sup> Tianbo Liu,<sup>3,\*</sup>  
 Raza Sabbir Sufian,<sup>4,5,†</sup> Stanley J. Brodsky,<sup>6</sup> and Alexandre Deur<sup>5</sup>  
 (HLFHS Collaboration)

$$\langle r_g^2 \rangle = \frac{6}{A^g(0)} \left. \frac{dA^g(t)}{dt} \right|_{t=0},$$

$$\langle r_g \rangle_p = 0.34 \text{ fm} \quad \langle r_g \rangle_\pi = 0.31 \text{ fm}$$

Momentum fraction carried by gluons in the proton:

$$A^g(0)_p = 0.225$$

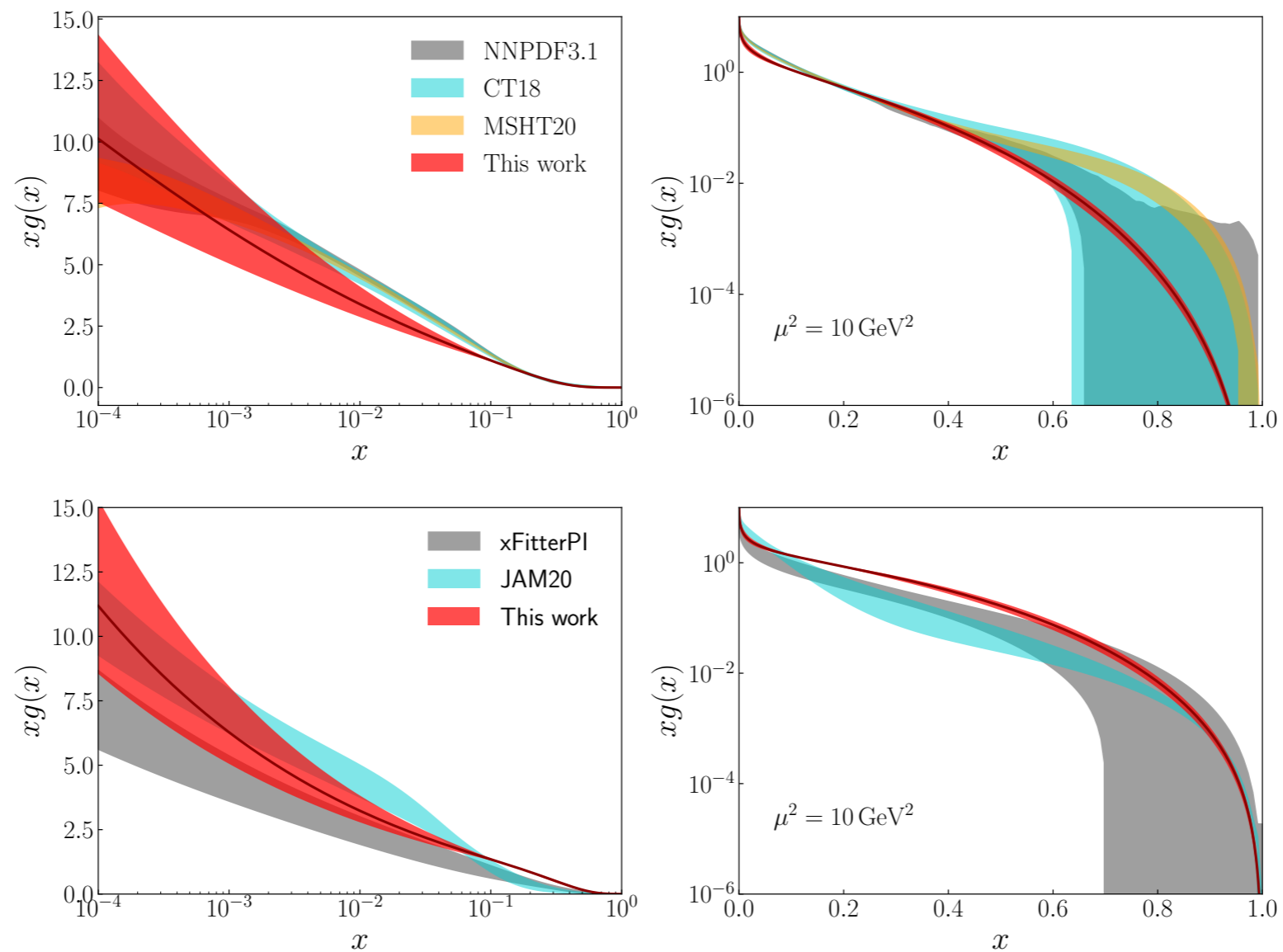
Momentum fraction carried by gluons in the pion:

$$A^g(0)_\pi = 0.429$$

*Compare with the gluonic radius*

The pion's electromagnetic radius is 0.657 fm

The proton's electromagnetic radius is 0.833 fm



Unpolarized gluon distribution in the proton (top panel) and pion (bottom panel) from HLFQCD and comparison with global fits. The figures on left and right are the same distributions with different scales for  $xg(x)$  and  $x$  to enhance the view of the small and large- $x$  regions respectively.

JLAB-THY-21-3454  
SLAC-PUB-17612

## Gluon matter distribution in the proton and pion from extended holographic light-front QCD

Guy F. de Téramond,<sup>1</sup> H. G. Dosch,<sup>2</sup> Tianbo Liu,<sup>3, \*</sup>  
Raza Sabbir Sufian,<sup>4, 5, †</sup> Stanley J. Brodsky,<sup>6</sup> and Alexandre Deur<sup>5</sup>  
(HLFHS Collaboration)

# Longitudinal dynamics and chiral symmetry breaking in holographic light-front QCD

Guy F. de Téramond<sup>1,\*</sup> and Stanley J. Brodsky<sup>2,†</sup>

<sup>1</sup>*Laboratorio de Física Teórica y Computacional, Universidad de Costa Rica, 11501 San José, Costa Rica*

<sup>2</sup>*SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94309, USA*

(Dated: April 18, 2021)

The breaking of chiral symmetry in holographic light-front QCD is encoded in its longitudinal dynamics with its chiral limit protected by the superconformal algebraic structure which governs its transverse dynamics. The scale in the longitudinal light-front Hamiltonian determines the confinement strength in this direction: It is also responsible for most of the light meson ground state mass consistent with the Gell-Mann-Oakes-Renner constraint. Longitudinal confinement and the breaking of chiral symmetry are found to be different manifestations of the same underlying dynamics like in 't Hooft large  $N_C$  QCD(1 + 1) model.

# Transverse and Longitudinal LF Confinement

$$M_H^2 = M_{||}^2 + M_{\perp}^2$$

$$\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U_{\perp}(\zeta) \right) \phi(\zeta) = M_{\perp}^2 \phi(\zeta),$$

$$\left( \frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} + U_{||}(x) \right) \chi(x) = M_{||}^2 \chi(x),$$

Longitudinal contribution for nonzero quark mass

S. S. Chabysheva and J.R.Hiller,

Constraint: Rotational symmetry in  
non-relativistic heavy-quark limit.



## Transverse Confinement

$$\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U_{\perp}(\zeta) \right) \phi(\zeta) = M_{\perp}^2 \phi(\zeta),$$

$$U_{\perp}(\zeta) = \lambda^2 \zeta^2 + 2\lambda(J - 1). \quad \zeta^2 = b_{\perp}^2 x(1 - x)$$

$$M_{\perp}^2(n, J, L) = 4\lambda \left( n + \frac{J + L}{2} \right),$$

and eigenfunctions

de Teramond, Dosch, sjb

$$\phi_{n,L}(\zeta) = \lambda^{(1+L)/2} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\lambda\zeta^2/2} L_n^L(\lambda\zeta^2)$$

$M_{\pi} = 0$  in chiral ( $m_q = 0$ ) limit

# Longitudinal Confinement

$$\left( \frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} + U_{||}(x) \right) \chi(x) = M_{||}^2 \chi(x)$$

$$U_{||}(x) = -\sigma^2 \partial_x (x(1-x) \partial_x) \quad \text{Li, Maris, Zhao, Vary}$$

$$U_{||} = \sigma^2 x(1-x) \tilde{z}^2$$

Ioffe length  $\tilde{z}$ : conjugate to LF  $x = \frac{k^+}{P^+}$  G.A. Miller, sjb

$\frac{\gamma^+ \gamma^+}{k^+^2}$  LF interaction in  $A^+ = 0$  gauge

de Teramond, sjb

Same potential: t' Hooft Equation QCD(1+1) $_{N_C \rightarrow \infty}$

# Longitudinal Confinement

$$U_{||} = \sigma^2 x(1-x)\tilde{z}^2$$

$$\left( \frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} \right) \chi(x) + \frac{g^2 N_C}{\pi} P \int_0^1 dx' \frac{\chi(x) - \chi(x')}{(x-x')^2}$$

$$\sigma = g \sqrt{\pi N_C / 3} = \text{const}, \quad = M_{||}^2 \chi(x),$$

$$\chi(x) \sim x^{\frac{2m_q}{\sigma}} (1-x)^{\frac{2m_{\bar{q}}}{\sigma}}$$

$$M_{\pi}^2 = g \sqrt{\pi N_C / 3} (m_u + m_d) + \mathcal{O}((m_u + m_d)^2)$$

GMOR relation

de Teramond, sjb

$$M_\pi^2 = \sigma(m_u + m_d) + \mathcal{O}((m_u + m_d)^2),$$

in the limit  $m_u, m_d \rightarrow 0$ . It has the same linear dependence in the quark mass as the Gell-Mann-Oakes-Renner (GMOR) relation

$$M_\pi^2 f_\pi^2 = -\frac{1}{2}(m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle + \mathcal{O}((m_u + m_d)^2)$$

where the “vacuum condensate”  $\langle \bar{\psi}\psi \rangle \equiv \frac{1}{2} \langle \bar{u}u + \bar{d}d \rangle$  plays the role of a chiral order parameter. The same linear dependence arises for the  $(3 + 1)$  effective LF Hamiltonian, since the constraints from the superconformal algebra require that the contribution to the pion mass from the transverse LF dynamics is identically zero.

Interpret  $\langle \bar{\psi}\psi \rangle$  as an *in-hadron condensate*

Expand in complete orthonormal basis

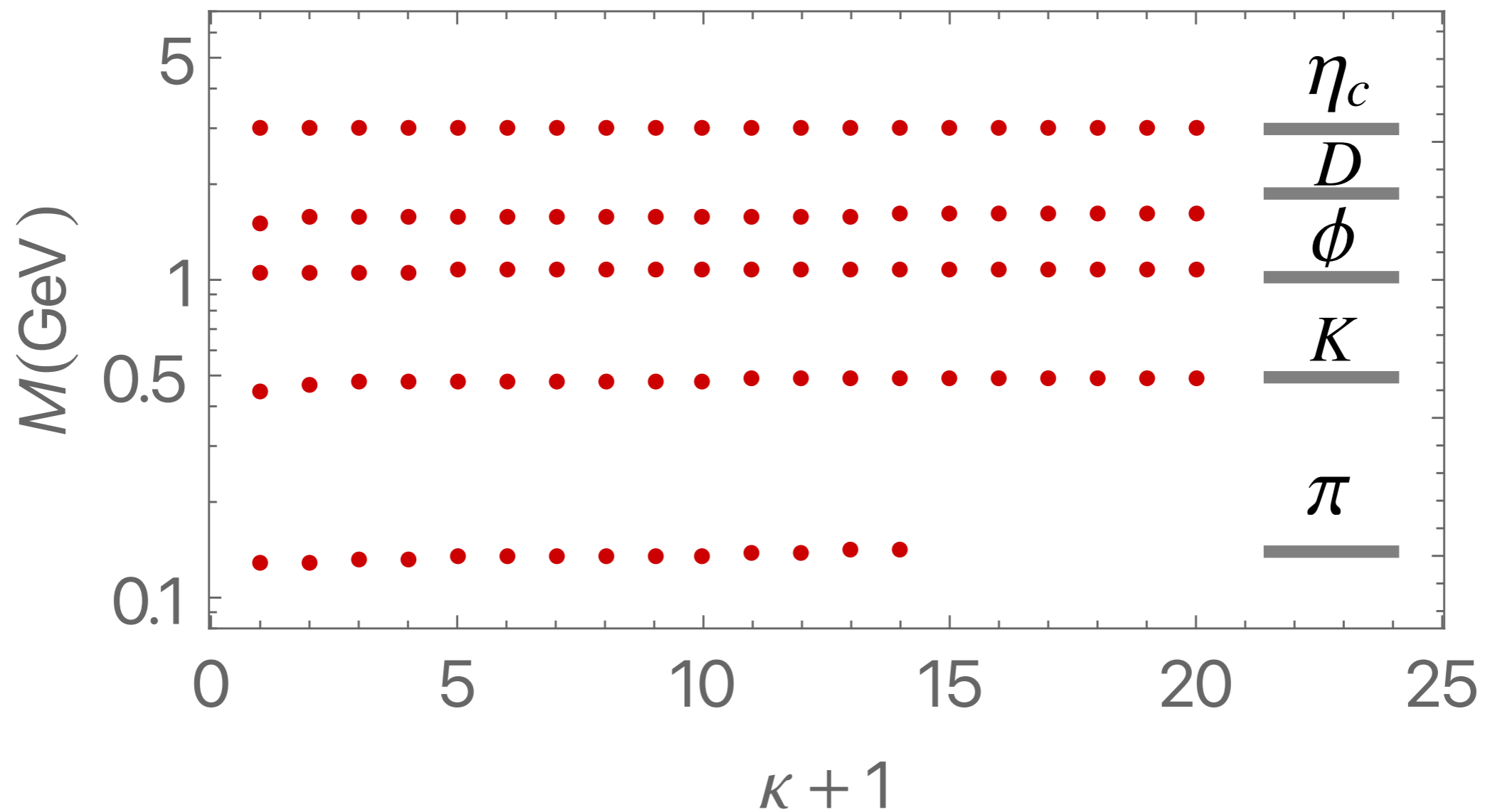
$$\chi_{\kappa}^{\alpha, \beta}(x) = N x^{\alpha/2} (1-x)^{\beta/2} P_{\kappa}^{(\alpha, \beta)}(1-2x).$$

$$M_{\parallel}^2 = \sigma^2 \int_0^1 dx \chi(x) \left( -\partial_x (x(1-x)\partial_x) + \frac{1}{4} \left[ \frac{\alpha^2}{x} + \frac{\beta^2}{1-x} \right] \right) \chi(x) = \sigma^2 \sum_{\kappa} C_{\kappa}^2 \nu^2(\kappa, \alpha, \beta),$$

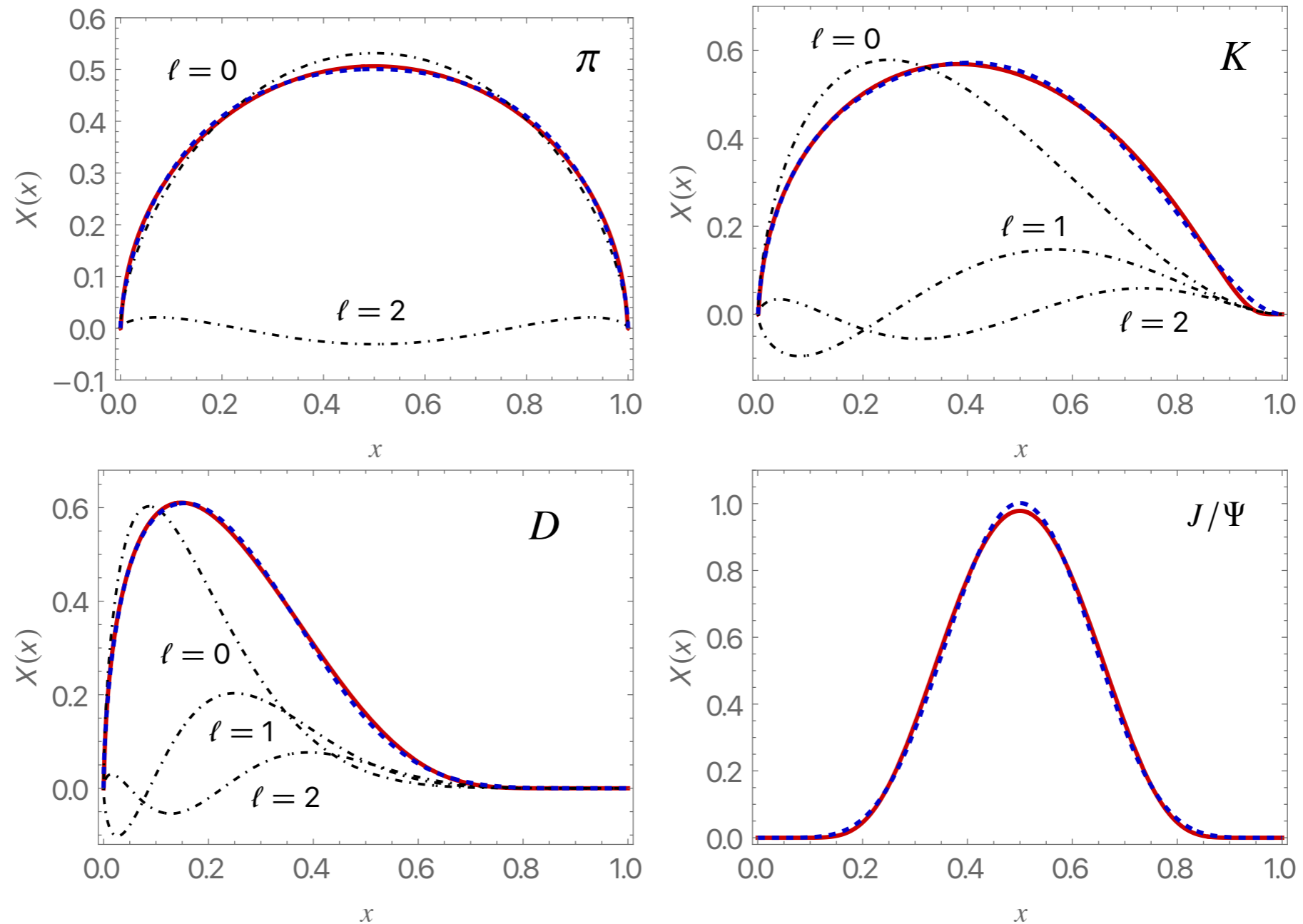
where  $\nu^2(\kappa, \alpha, \beta) = \frac{1}{4}(\alpha + \beta + 2\kappa)(2 + \alpha + \beta + 2\kappa)$ , with  $\alpha = 2m_q/\sigma$  and  $\beta = 2m_{\bar{q}}/\sigma$ .

*Mode expansion*





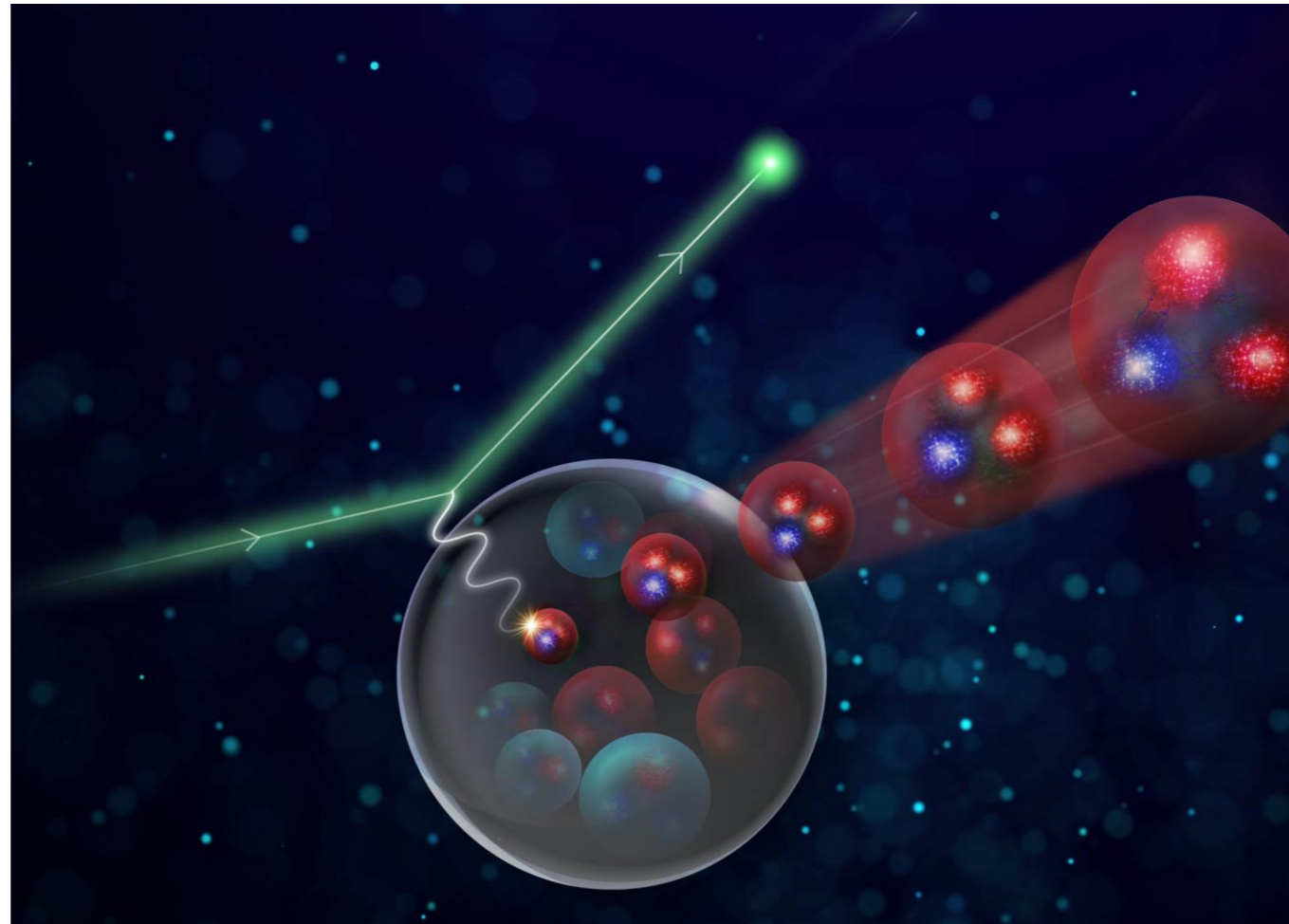
Convergence of ground state meson masses with increasing  $\kappa$   
 The horizontal grey lines in the figure are the observed masses.



Light-front distribution amplitudes  $X(x)$  for the  $\pi$ ,  $K$ ,  $D$  and  $J/\Psi$  mesons: the red curve is the invariant mass result, dot dashed black curves are individual modes in the expansion (16), dashed blue curve represent the sum of modes in the figure. Notice that the  $J/\Psi$  result is well described by the zero mode alone.

# The Onset of Color Transparency in Holographic Light-Front QCD

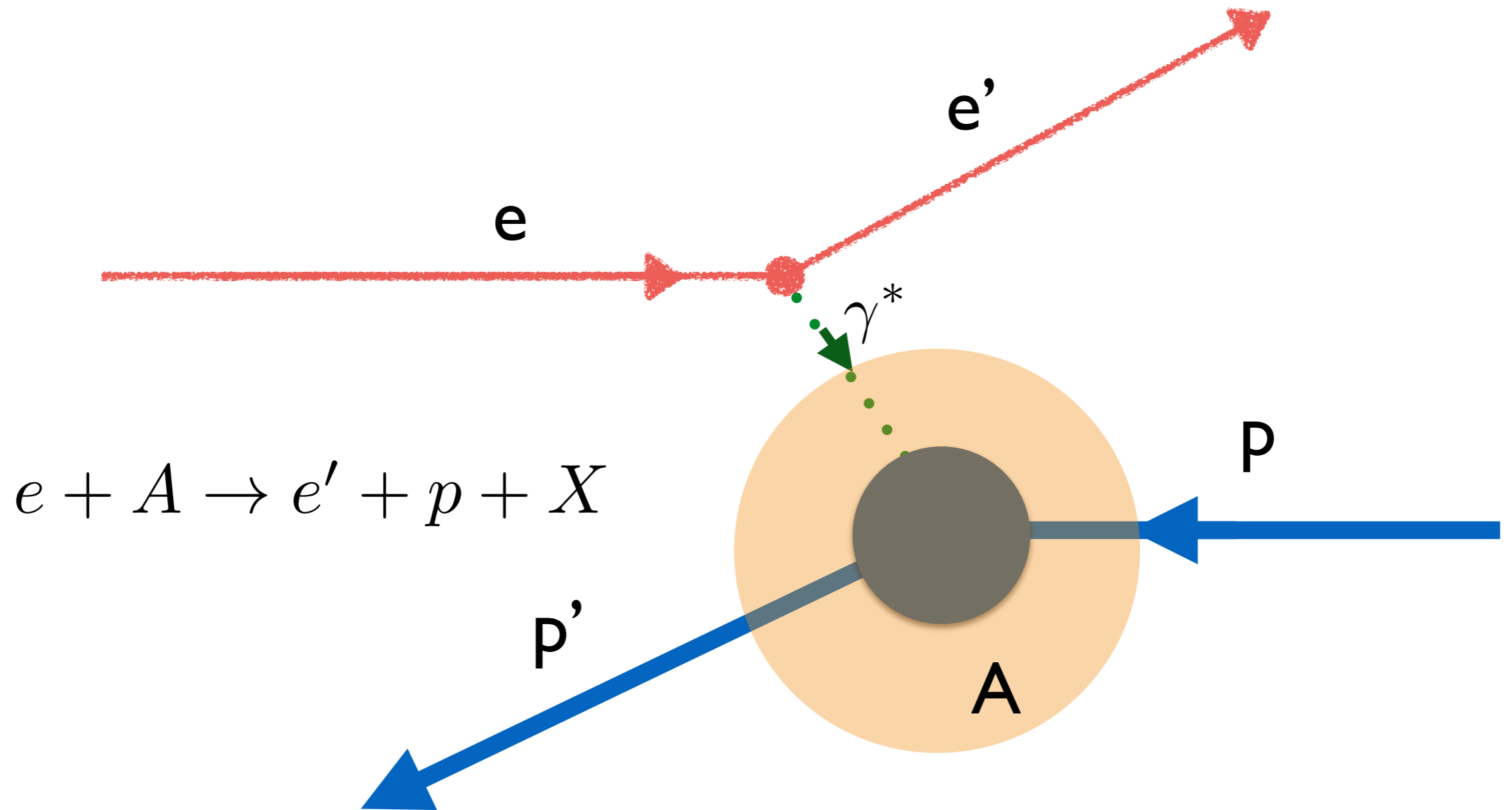
c



with Guy F. de Téramond

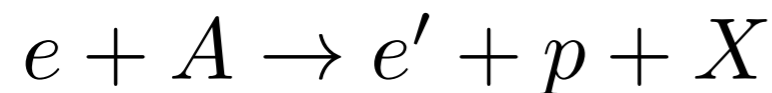
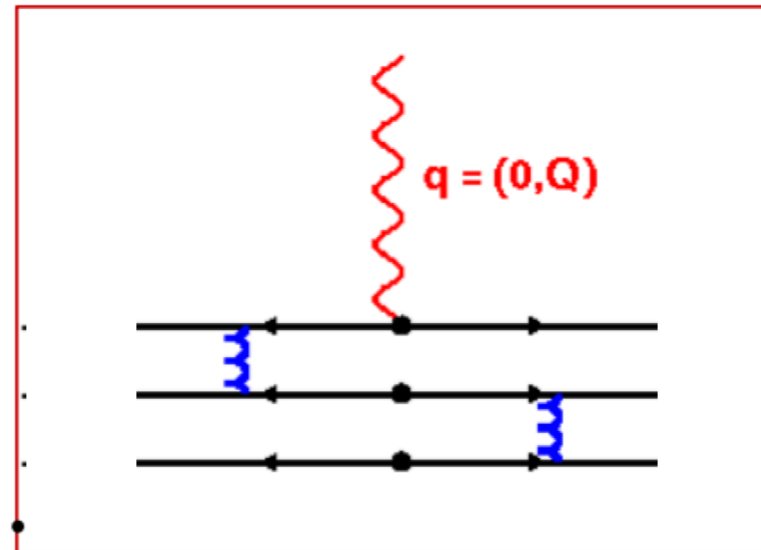
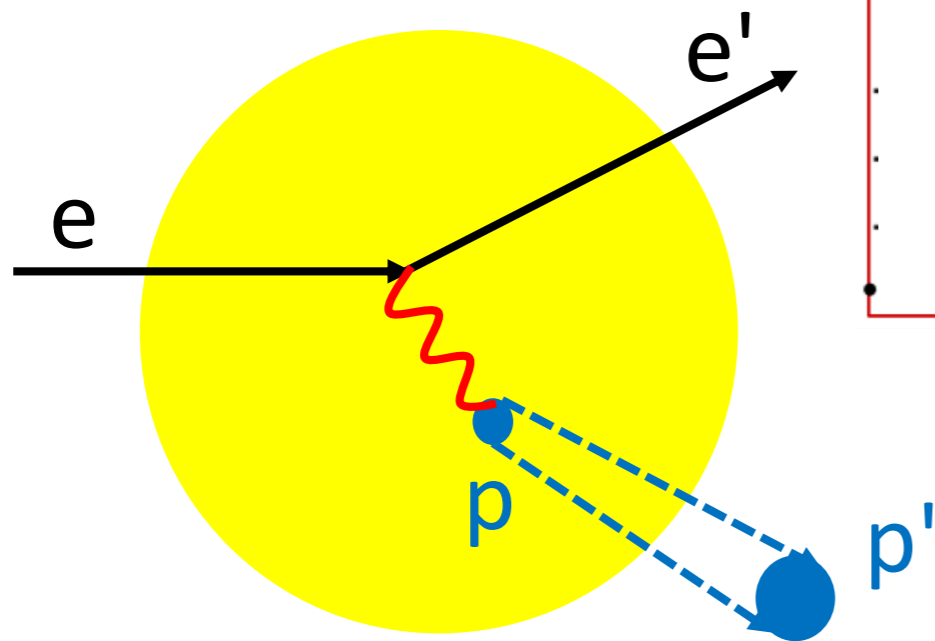
# Color Transparency

$$\sigma(e + A \rightarrow e' + p + X) \rightarrow Z \frac{d\sigma}{dt}(ep \rightarrow e'p') \quad \text{at high } Q^2$$



- **QCD: Gauge theory properties and quantum coherence**
- **Small-size color dipole moment interacts weakly in nuclei**

# Color transparency fundamental prediction of QCD

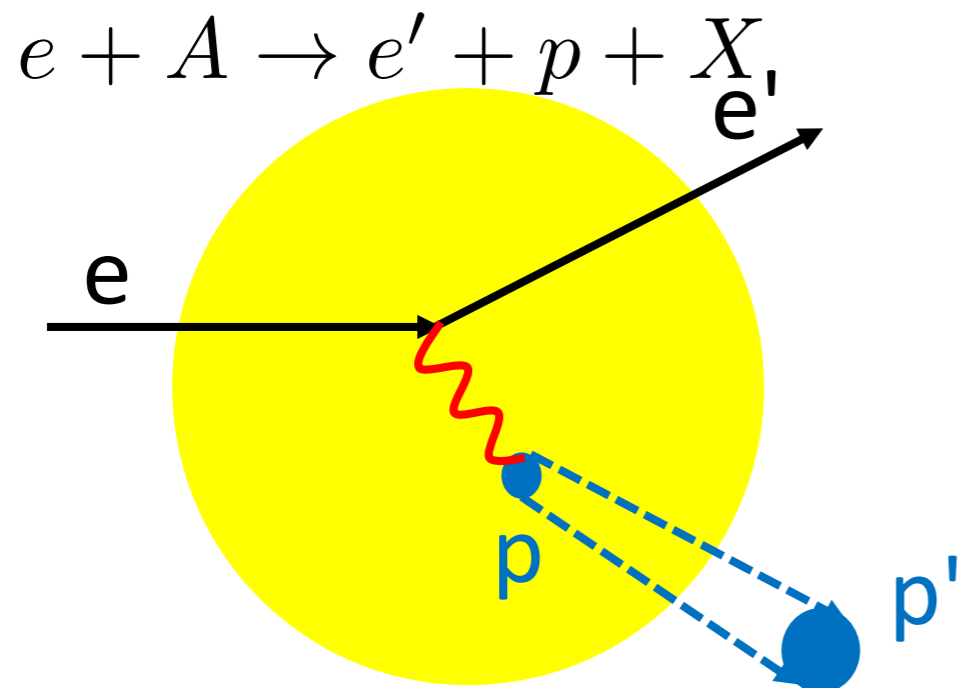


- Introduced by Mueller and Brodsky, 1982
- Vanishing of initial/final state interaction of hadrons with nuclear medium in exclusive processes at high momentum transfer

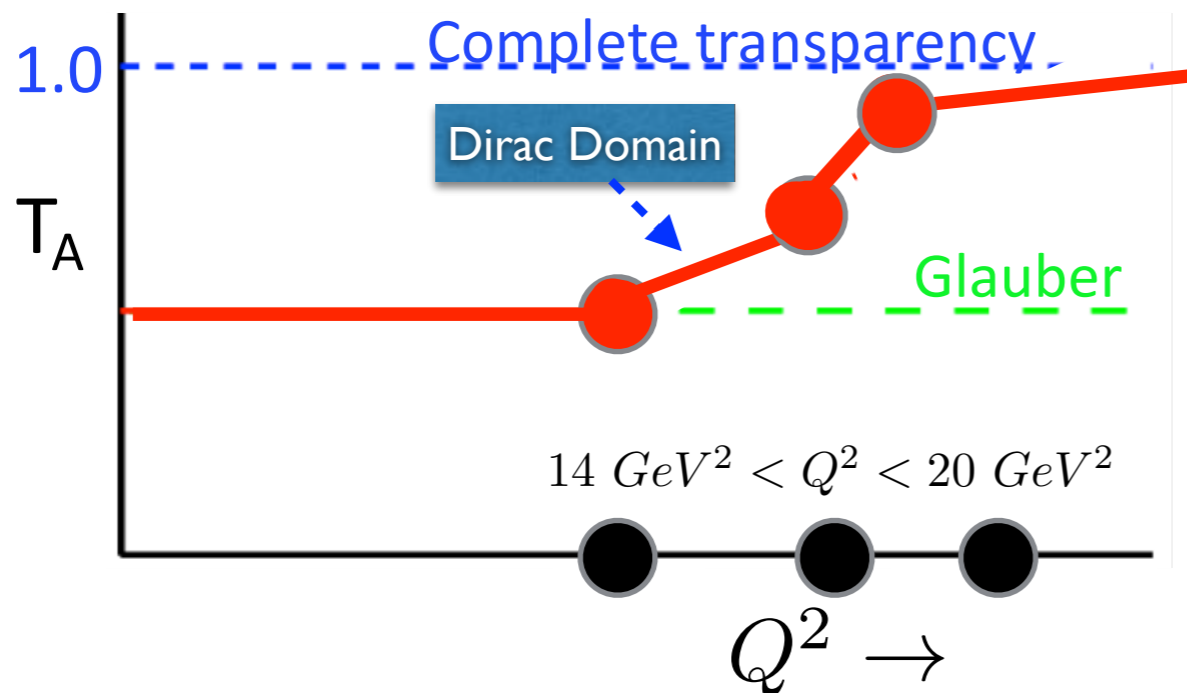
- Hadron fluctuates to small transverse size ([quantum mechanics](#))
- Maintains this small size as it propagates out of the nucleus ([relativity](#))
- Experiences reduced attenuation in nucleus, color screened ([strong force](#))



# Color transparency fundamental prediction of QCD



- Not predicted by strongly interacting hadronic picture  $\rightarrow$  arises in picture of quark-gluon interactions
- QCD: color field of singlet objects vanishes as size is reduced
- Signature is a rise in nuclear transparency,  $T_A$ , as a function of the momentum transfer,  $Q^2$



$$T_A = \frac{\sigma_A}{A \sigma_N} \quad \begin{array}{l} \text{(nuclear cross section)} \\ \text{(free nucleon cross section)} \end{array}$$

# Color Transparency

**Mueller, sjb**

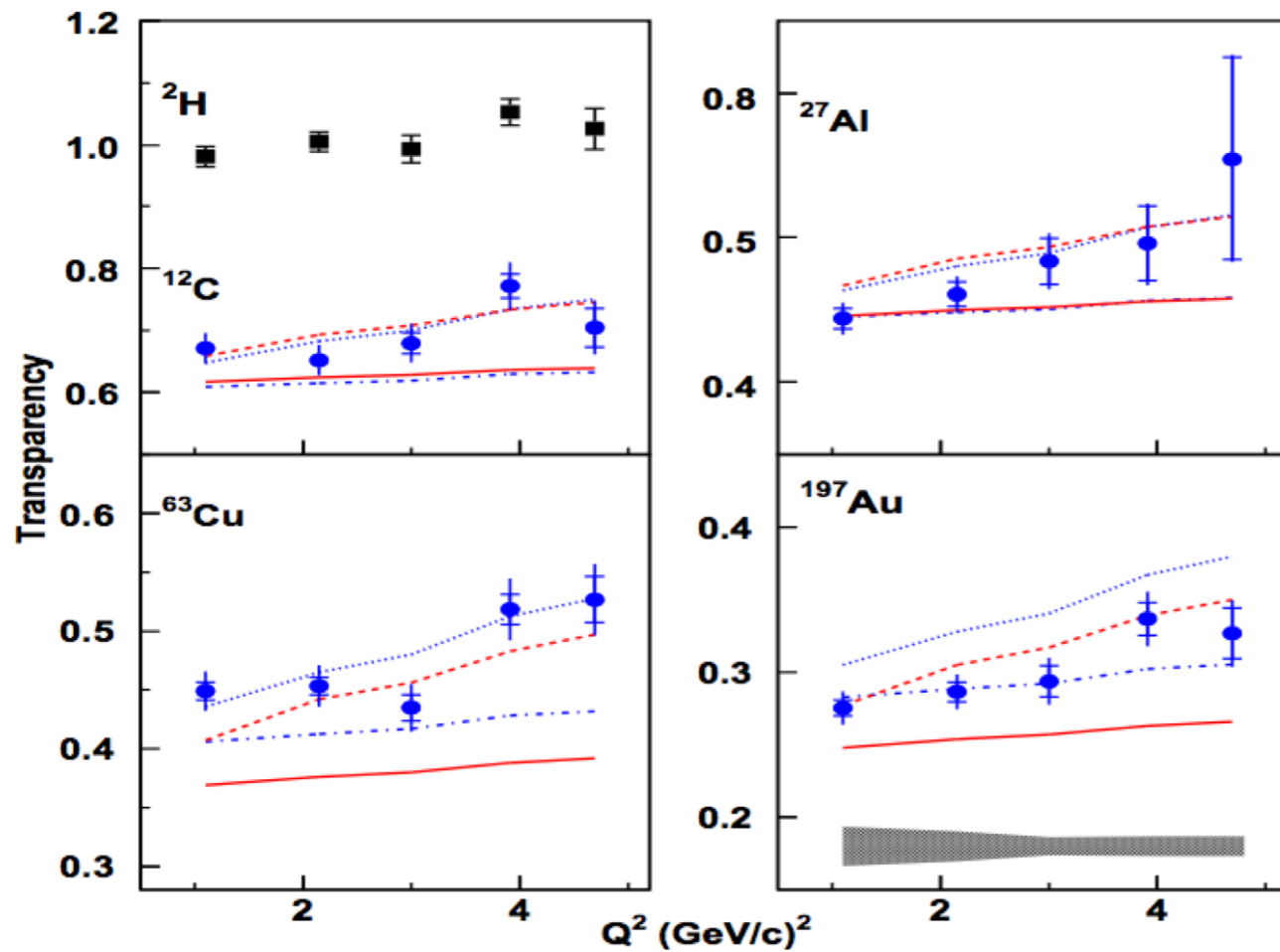
**Bertsch, Gunion, Goldhaber, sjb**

$$\frac{d\sigma}{dt}(eA \rightarrow ep(A-1)) = Z \frac{d\sigma}{dt}(ep \rightarrow ep) \quad \text{at high momentum transfer}$$

- **Fundamental test of gauge theory in hadron physics**
- **Small color dipole moment interacts weakly in nuclei**
- **Complete coherence at high energies**
- **Many tests in hard exclusive processes**
- **Clear Demonstration of CT from Diffractive Di-Jets**
- **Explains Baryon Anomaly at RHIC**
- **Small color dipole moment interacts weakly in nuclei**

Hall C E01-107 pion electro-production

$$A(e, e' \pi^+)$$

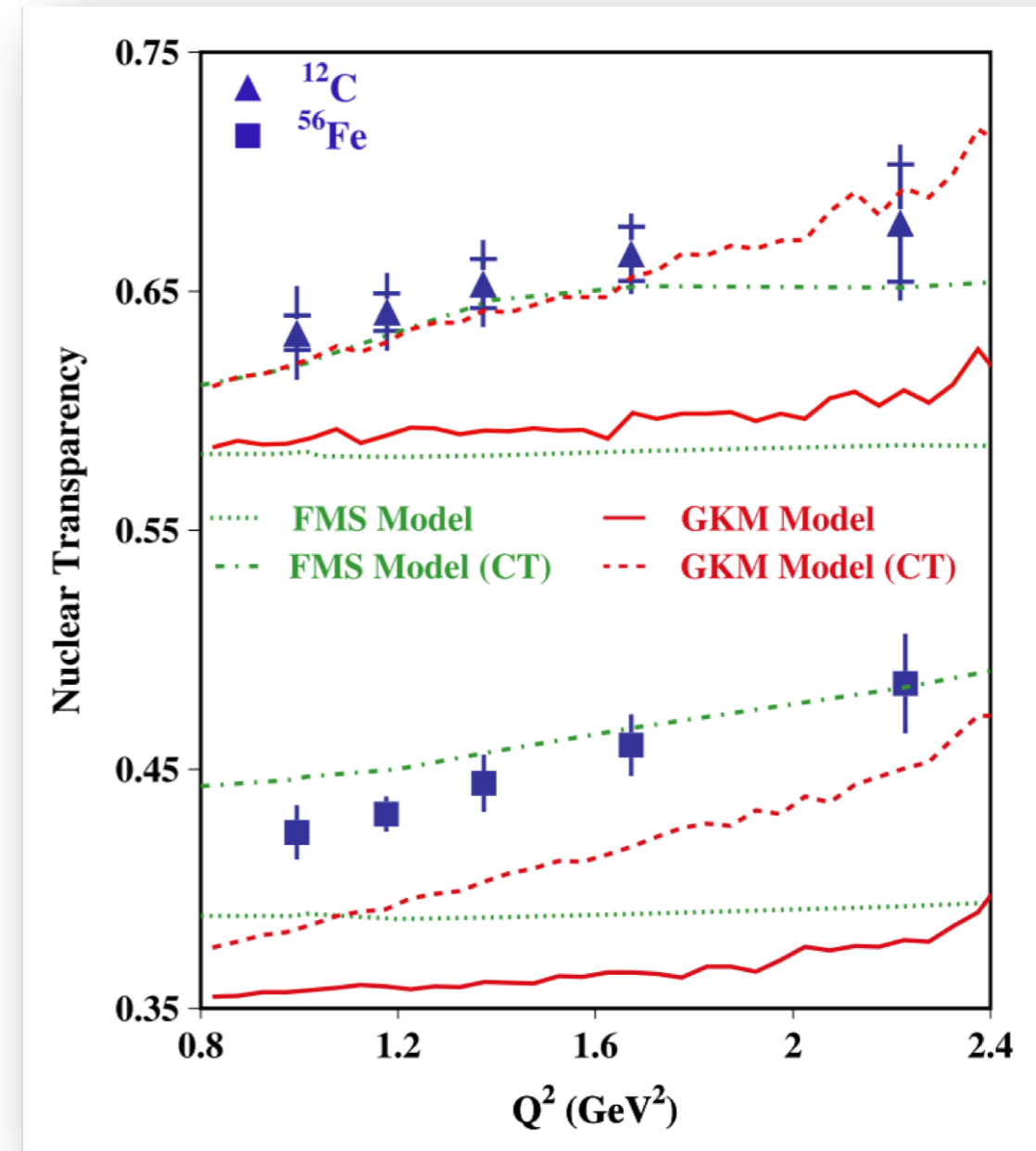


B. Clasie *et al.* PRL 99:242502 (2007)

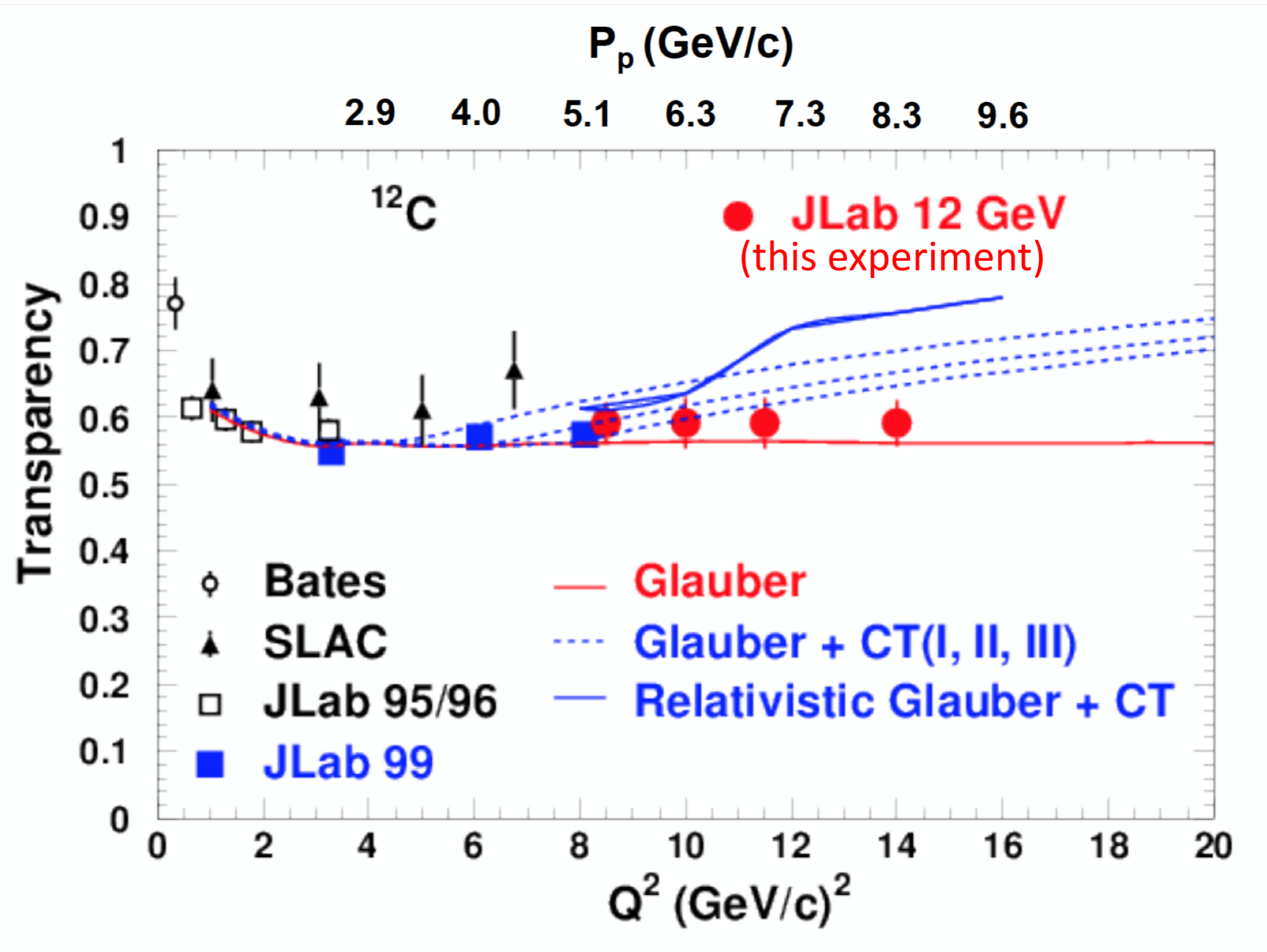
X. Qian *et al.* PRC81:055209 (2010)

CLAS E02-110 rho electro-production

$$A(e, e' \rho^0)$$



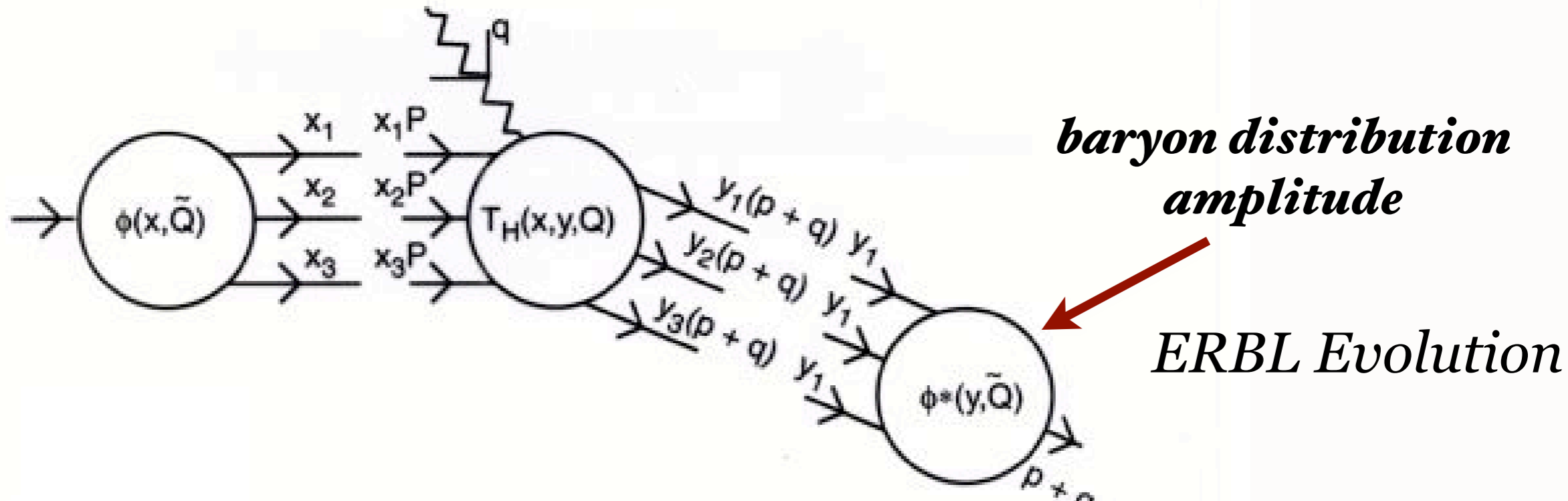
L. El Fassi *et al.* PLB 712,326 (2012)



Ruling out color transparency in quasi-elastic  $^{12}\text{C}(e,e'p)$  up to  $Q^2$  of 14.2 (GeV/c)<sup>2</sup>  
Hall C Collaboration

# Leading-Twist PQCD Factorization for form factors, exclusive amplitudes

Lepage, sjb

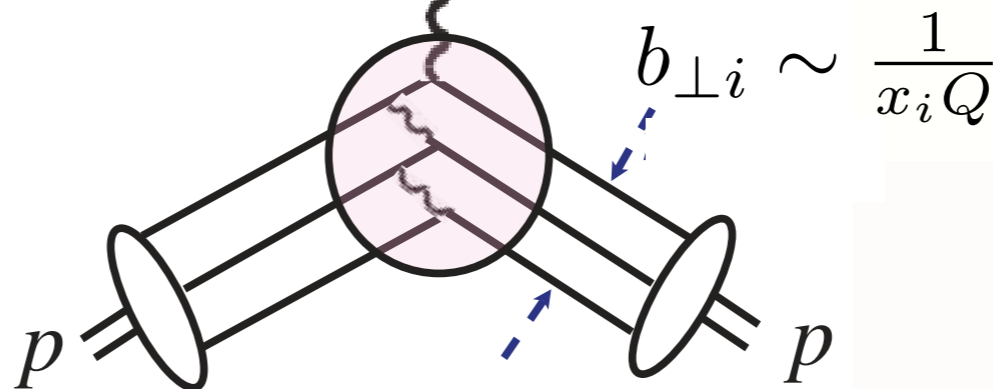


$$M = \int \prod dx_i dy_i \phi_F(x_i, \tilde{Q}) \times T_H(x_i, y_i, \tilde{Q}) \times \phi_I(y_i, \tilde{Q})$$

Exclusive

Scaling Laws  
Counting Rules

$$\langle x_i \rangle \sim \frac{1}{n} = \frac{1}{\text{twist } \tau}$$

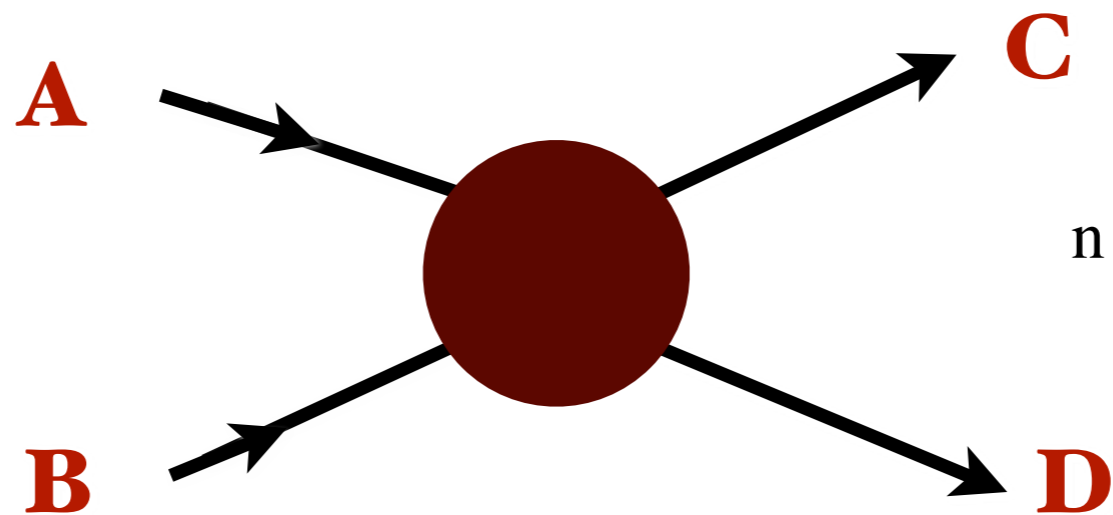




# “Counting Rules” *Farrar and sjb; Muradyan, Matveev, Tavkelidze*

$$\frac{d\sigma}{dt}(A + B \rightarrow C + D) = \frac{F(t/s)}{s^{n_{tot}-2}}$$

$$n_{tot} = n_A + n_B + n_C + n_D$$



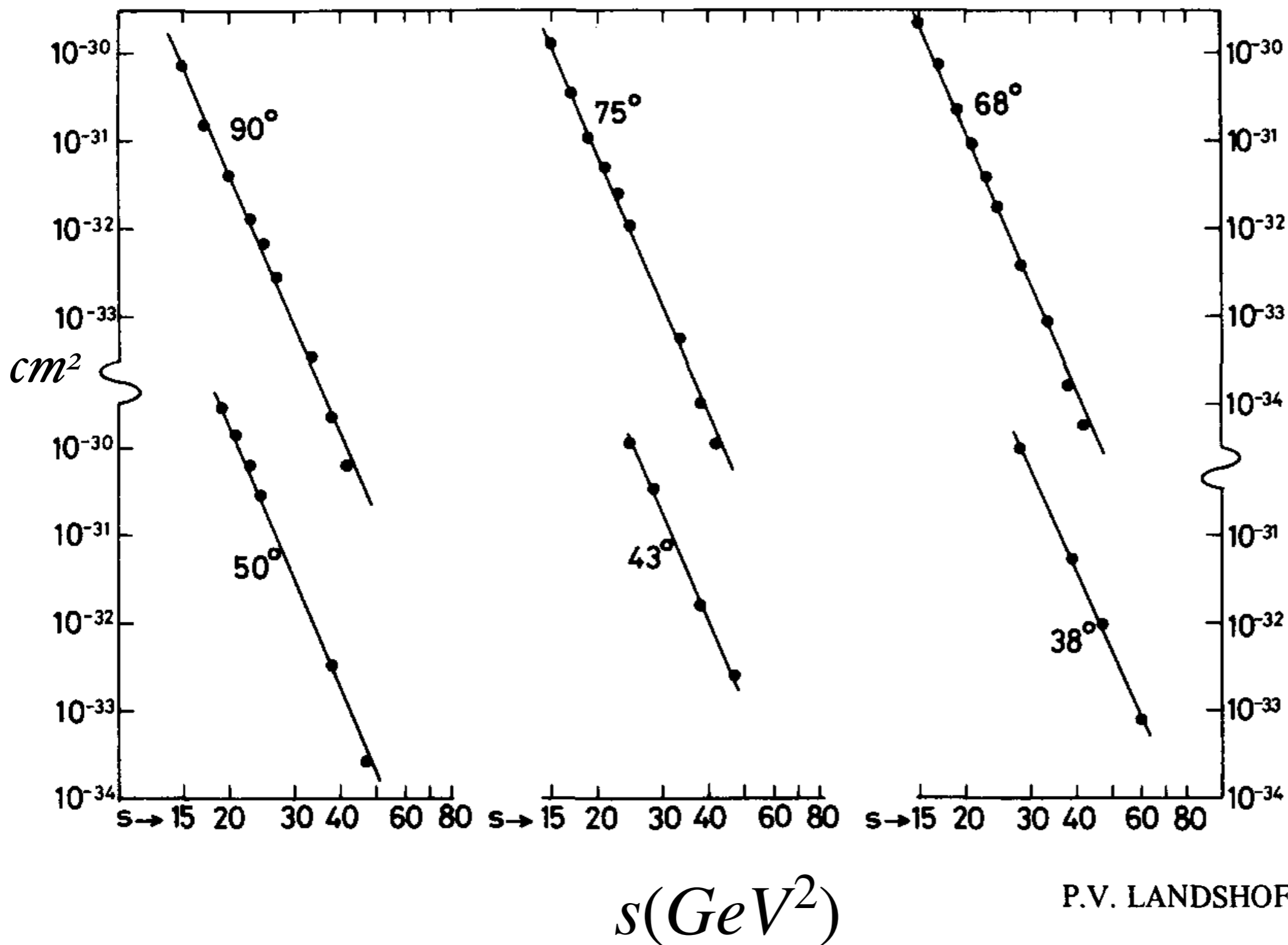
$n = \text{twist } \tau = \text{dimension} - \text{spin}$

e.g.  $n_{tot} - 2 = n_A + n_B + n_C + n_D - 2 = 10$  for  $pp \rightarrow pp$

*Predict:* 
$$\frac{d\sigma}{dt}(p + p \rightarrow p + p) = \frac{F(\theta_{CM})}{s^{10}}$$

*Quark-Counting* :  $\frac{d\sigma}{dt}(pp \rightarrow pp) = \frac{F(\theta_{CM})}{s^{10}}$

$$n = 4 \times 3 - 2 = 10$$



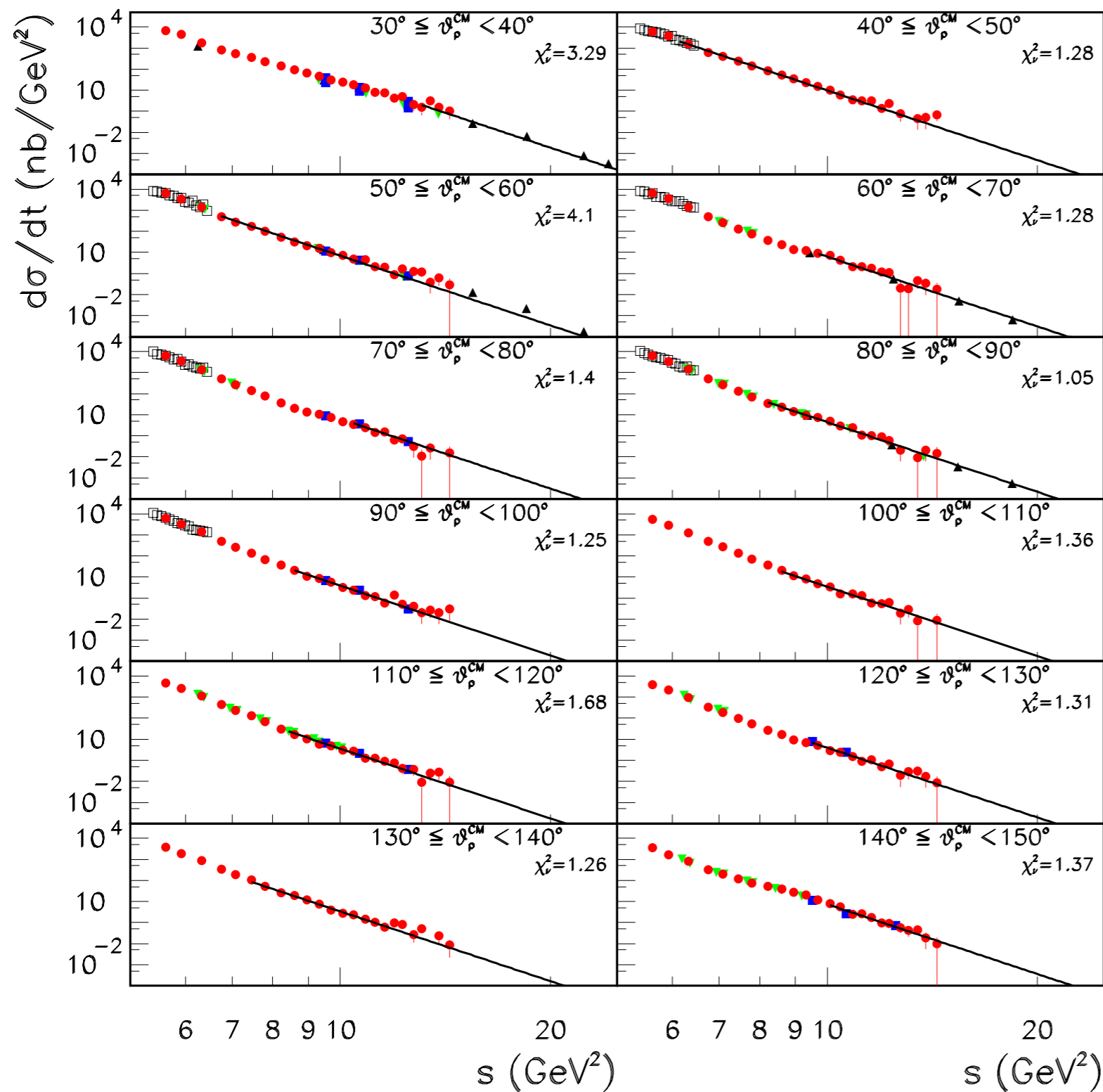
*Best Fit*

$$n = 9.7 \pm 0.5$$

*Reflects underlying conformal, scale-free interactions*

P.V. LANDSHOFF and J.C. POLKINGHORNE

# Deuteron Photodisintegration & Dimensional Counting Rules



$$s^{n_{tot}-2} \frac{d\sigma}{dt}(A+B \rightarrow C+D) = F_{A+B \rightarrow C+D}(\theta_{CM})$$

$$s^{11} \frac{d\sigma}{dt}(\gamma d \rightarrow np) = F(\theta_{CM})$$

$$n_{tot} - 2 = (1 + 6 + 3 + 3) - 2 = 11$$

$$F_D(Q^2) \sim \left[\frac{1}{Q^2}\right]^5$$

# Scaling is a manifestation of asymptotically free hadron interactions

From dimensional arguments at high energies in binary reactions:

## CONSTITUENT COUNTING RULES

Brodsky and Farrar, Phys. Rev. Lett. 31 (1973) 1153  
Matveev et al., Lett. Nuovo Cimento, 7 (1973) 719

Counting Rules:

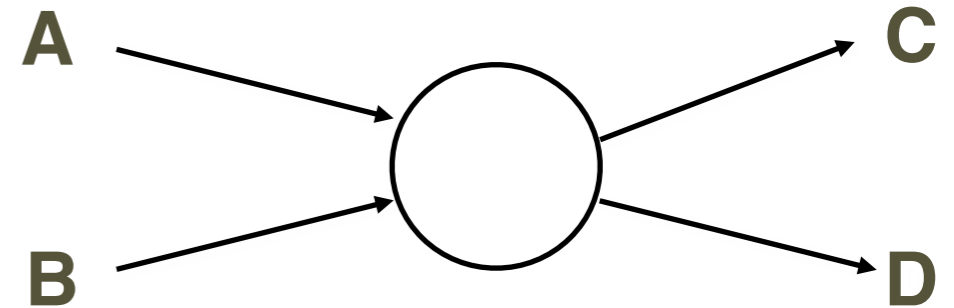
$$q(x) \sim (1-x)^{2n_{spect}-1} \text{ for } x \rightarrow 1$$

$$F(Q^2) \sim \left(\frac{1}{Q^2}\right)^{(n-1)}$$

$$\frac{d\sigma}{dt}(AB \rightarrow CD) \sim \frac{F(t/s)}{s^{(n_{participants}-2)}}$$

$$n_{participants} = n_A + n_B + n_C + n_D$$

$$\frac{d\sigma}{d^3p/E}(AB \rightarrow CX) \sim F(\hat{t}/\hat{s}) \times \frac{(1-x_R)^{(2n_{spectators}-1)}}{(p_T^2)^{(n_{participants}-2)}}$$



Exclusive-Inclusive Connection  
Gribov-Lipatov crossing

## Drell-Yan-West Formula in Impact Space

$$\begin{aligned}
 F(q^2) &= \sum_n \prod_{i=1}^n \int dx_i \int \frac{d^2 \mathbf{k}_{\perp i}}{2(2\pi)^3} 16\pi^3 \delta\left(1 - \sum_{j=1}^n x_j\right) \delta^{(2)}\left(\sum_{j=1}^n \mathbf{k}_{\perp j}\right) \\
 &\quad \sum_j e_j \psi_n^*(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_n(x_i, \mathbf{k}_{\perp i}, \lambda_i), \\
 &= \sum_n \prod_{i=1}^{n-1} \int dx_j \int d^2 \mathbf{b}_{\perp j} \exp\left(i \mathbf{q}_{\perp} \cdot \sum_{i=1}^{n-1} x_j \mathbf{b}_{\perp j}\right) |\psi_n(x_j, \mathbf{b}_{\perp j})|^2
 \end{aligned}$$

$$\sum_{i=1}^n x_i = 1 \text{ and } \sum_{i=1}^n \mathbf{b}_{\perp i} = 0.$$

$$F(q^2) = \int_0^1 dx \int d^2 \mathbf{a}_{\perp} e^{i \mathbf{a}_{\perp} \cdot \mathbf{q}_{\perp}} q(x, \mathbf{a}_{\perp}),$$

where  $\mathbf{a}_{\perp} = \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}$  is the  $x$ -weighted transverse position coordinate of the  $n - 1$  spectators.



# Universality of Generalized Parton Distributions in Light-Front Holographic QCD

Guy F. de Téramond, Tianbo Liu, Raza Sabbir Sufian, Hans Günter Dosch, Alexandre Deur, SJB

(HLFHS Collaboration)

$$F_\tau(t) = \frac{1}{N_\tau} B\left(\tau - 1, \frac{1}{2} - \frac{t}{4\lambda}\right), \quad N_\tau = B(\tau - 1, 1 - \alpha(0))$$

$$B(u, v) = \int_0^1 dy y^{u-1} (1-y)^{v-1} = [\Gamma(u)\Gamma(v)/\Gamma(u+v)]$$

$$F_\tau(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_0^2}\right)\left(1 + \frac{Q^2}{M_1^2}\right) \cdots \left(1 + \frac{Q^2}{M_{\tau-2}^2}\right)}$$

$$F_\tau(Q^2) \sim \left(\frac{1}{Q^2}\right)^{\tau-1}$$

$$M_n^2 = 4\lambda\left(n + \frac{1}{2}\right), n = 0, 1, 2, \dots, \tau - 2, \quad M_0 = m_\rho$$

$$\sqrt{\lambda} = \kappa = \frac{m_\rho}{\sqrt{2}} = 0.548 \text{ GeV} \quad \frac{1}{2} - \frac{t}{4\lambda} = 1 - \alpha_R(t)$$

$\alpha_R(t) = \rho$  Regge Trajectory

$$F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j \int d^2 \mathbf{b}_{\perp j} \exp\left(i \mathbf{q}_{\perp} \cdot \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\right) |\psi_n(x_j, \mathbf{b}_{\perp j})|^2$$

$$\sum_i x_i = 1$$

$$\vec{a}_{\perp} \equiv \sum_{j=1}^{n-1} x_j \vec{b}_{\perp j}$$

$$\vec{a}_{\perp}^2(Q^2) = -4 \frac{\frac{d}{dQ^2} F(Q^2)}{F(Q^2)}$$

Proton radius squared at  $Q^2 = 0$

Color Transparency is controlled by the transverse-spatial size  $\vec{a}_{\perp}^2$  and its dependence on the momentum transfer  $Q^2 = -t$  :

Light-Front Holography:

For large  $Q^2$  :

$$\langle \mathbf{a}_{\perp}^2(t) \rangle_{\tau} = \frac{1}{\lambda} \sum_{j=1}^{\tau-1} \frac{1}{j - \alpha(t)}$$

$$\langle \mathbf{a}_{\perp}^2(Q^2) \rangle_{\tau} \rightarrow \frac{4(\tau - 1)}{Q^2}$$

The scale  $Q_{\tau}^2$  required for Color Transparency grows with twist  $\tau$

$$F(q^2) = \int_0^1 dx \int d^2 \mathbf{a}_\perp e^{i \mathbf{a}_\perp \cdot \mathbf{q}_\perp} q(x, \mathbf{a}_\perp)$$

Counting rules:

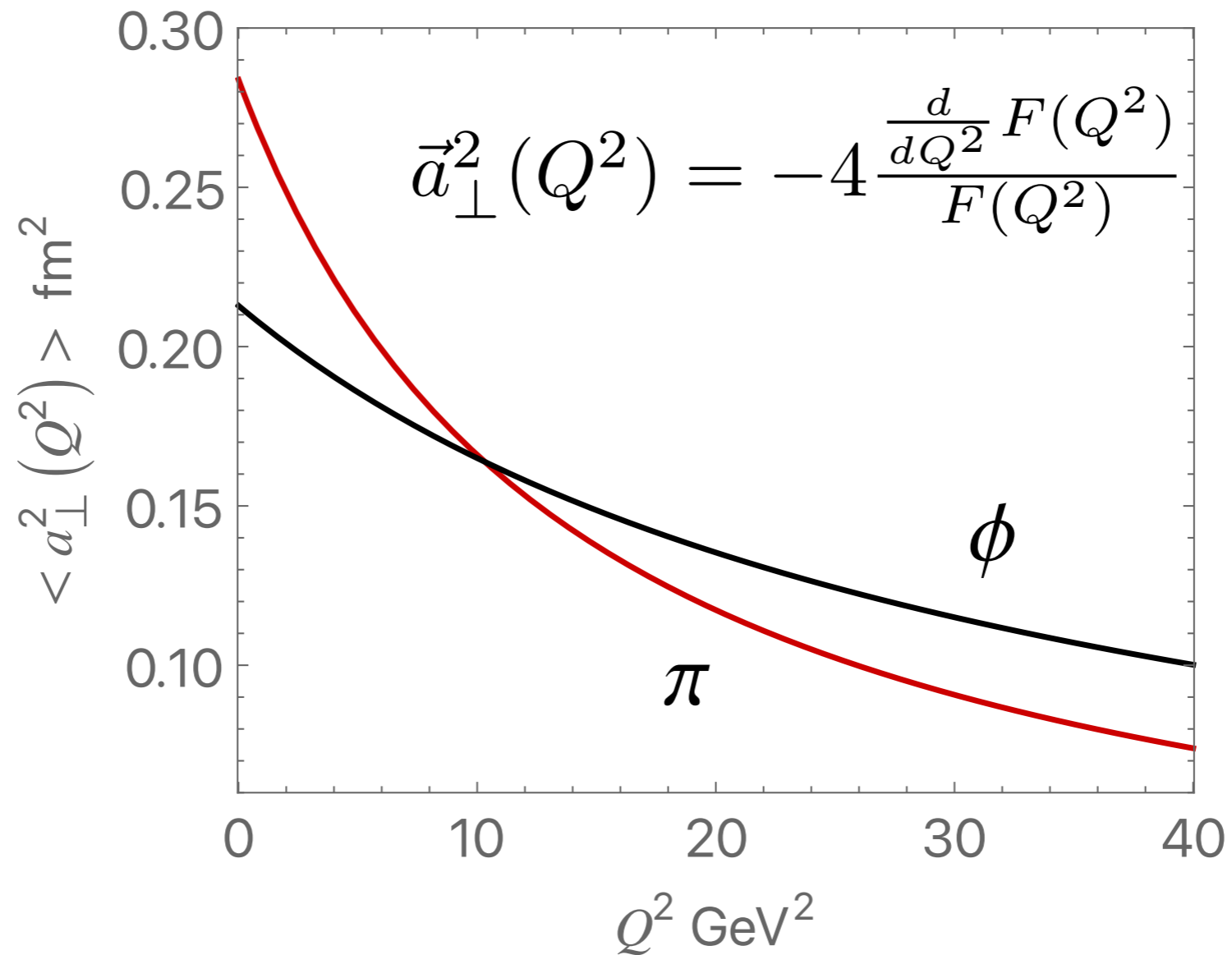
$$q_\tau(x) \propto (1-x)^{2n_s-1} = (1-x)^{2\tau-3}$$

$$\langle \mathbf{a}_\perp^2(Q^2) \rangle_\tau \rightarrow \frac{4(\tau-1)}{Q^2}$$

Transverse size  $a_\perp$  decreases with momentum transfer  $Q$ , grows with the number of spectators plus the internal orbital angular momentum  $L$

$$\text{Twist } \tau = n + L$$

$$\langle \mathbf{a}_{\perp}^2(t) \rangle_{\tau} = \frac{1}{\lambda} \sum_{j=1}^{\tau-1} \frac{1}{j - \alpha(t)},$$



Transverse size depends on internal dynamics

Transparency controlled by transverse size

$$F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j \int d^2 \mathbf{b}_{\perp j} \exp\left(i \mathbf{q}_{\perp} \cdot \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\right) |\psi_n(x_j, \mathbf{b}_{\perp j})|^2$$

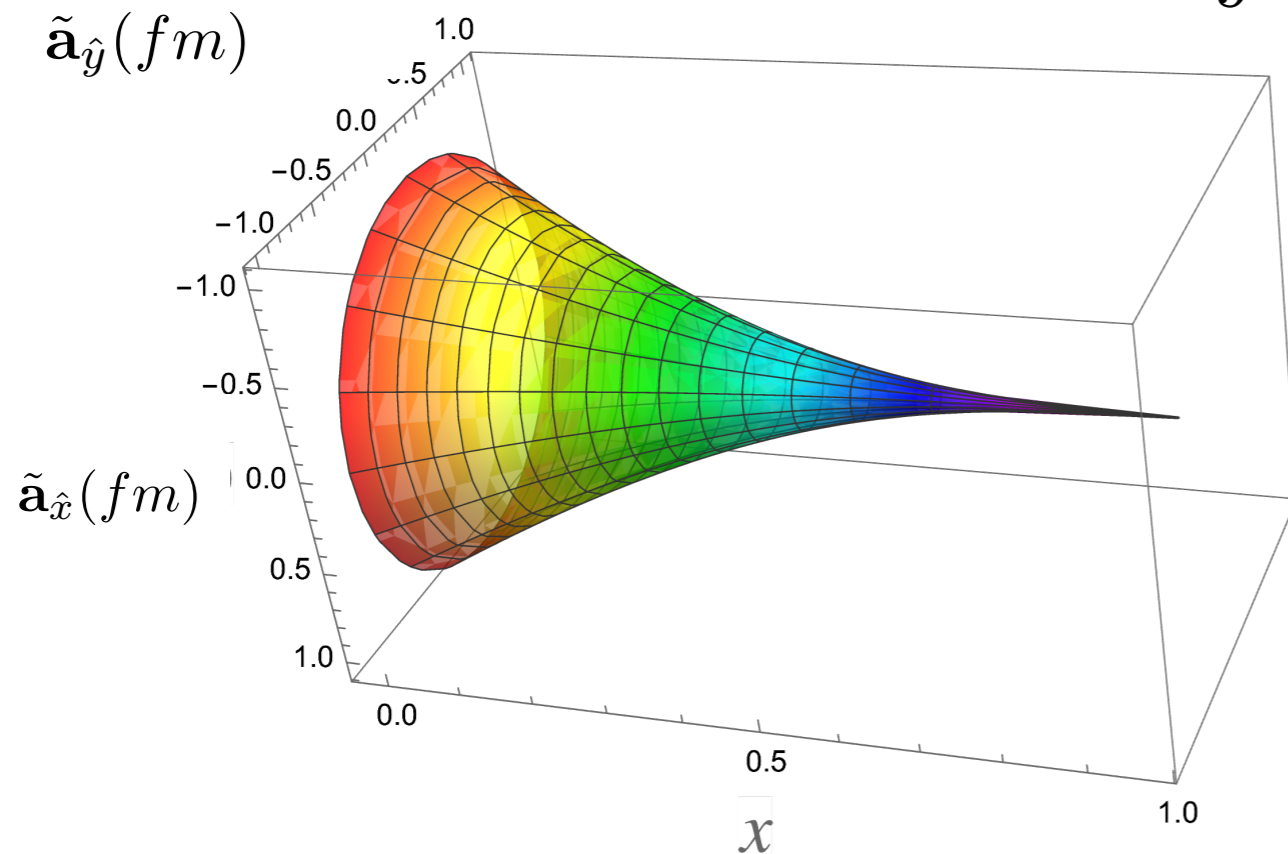
$$\vec{a}_{\perp} \equiv \sum_{j=1}^{n-1} x_j \vec{b}_{\perp j}$$

$$F(q^2) = \int_0^1 dx \int d^2 \mathbf{a}_{\perp} e^{i \mathbf{a}_{\perp} \cdot \mathbf{q}_{\perp}} q(x, \mathbf{a}_{\perp}),$$

$$x = 1 - \sum_{j=1}^{n-1} x_j$$

Define mean transverse size as a function of  $x$

$$\sigma(x) = \langle \tilde{\mathbf{a}}_{\perp}^2(x) \rangle = \frac{\int d^2 \mathbf{a}_{\perp} \mathbf{a}_{\perp}^2 q(x, \mathbf{a}_{\perp})}{\int d^2 \mathbf{a}_{\perp} q(x, \mathbf{a}_{\perp})}$$



$\langle \tilde{\mathbf{a}}_{\perp}^2(x) \rangle$ : averaged over  $Q^2$

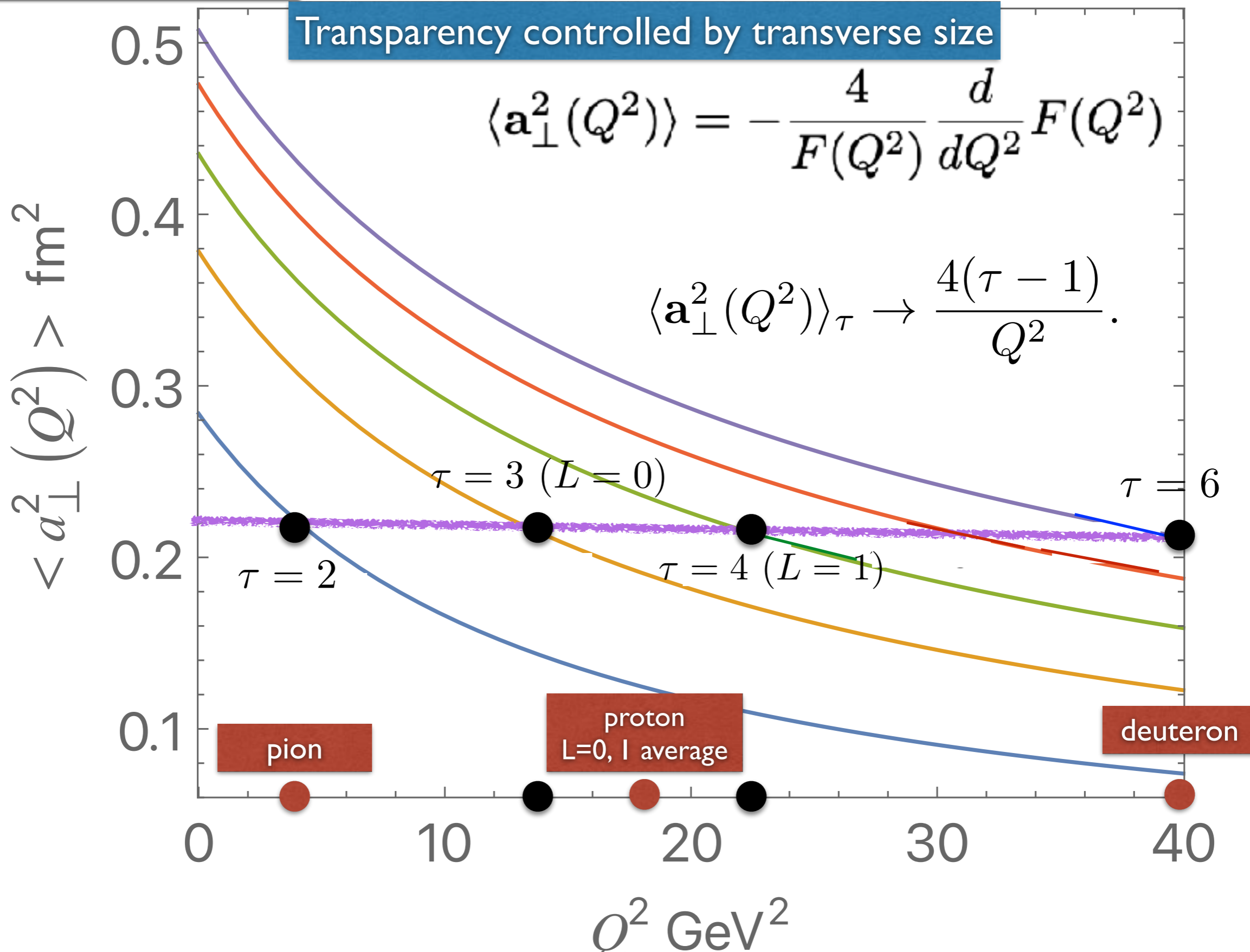
Mean transverse size as a function of  $Q$  and Twist

$$\langle \mathbf{a}_{\perp}^2(Q^2) \rangle_{\tau} \rightarrow \frac{4(\tau - 1)}{Q^2}.$$



Transparency scale  $Q$   
increases with twist

Transparency controlled by transverse size

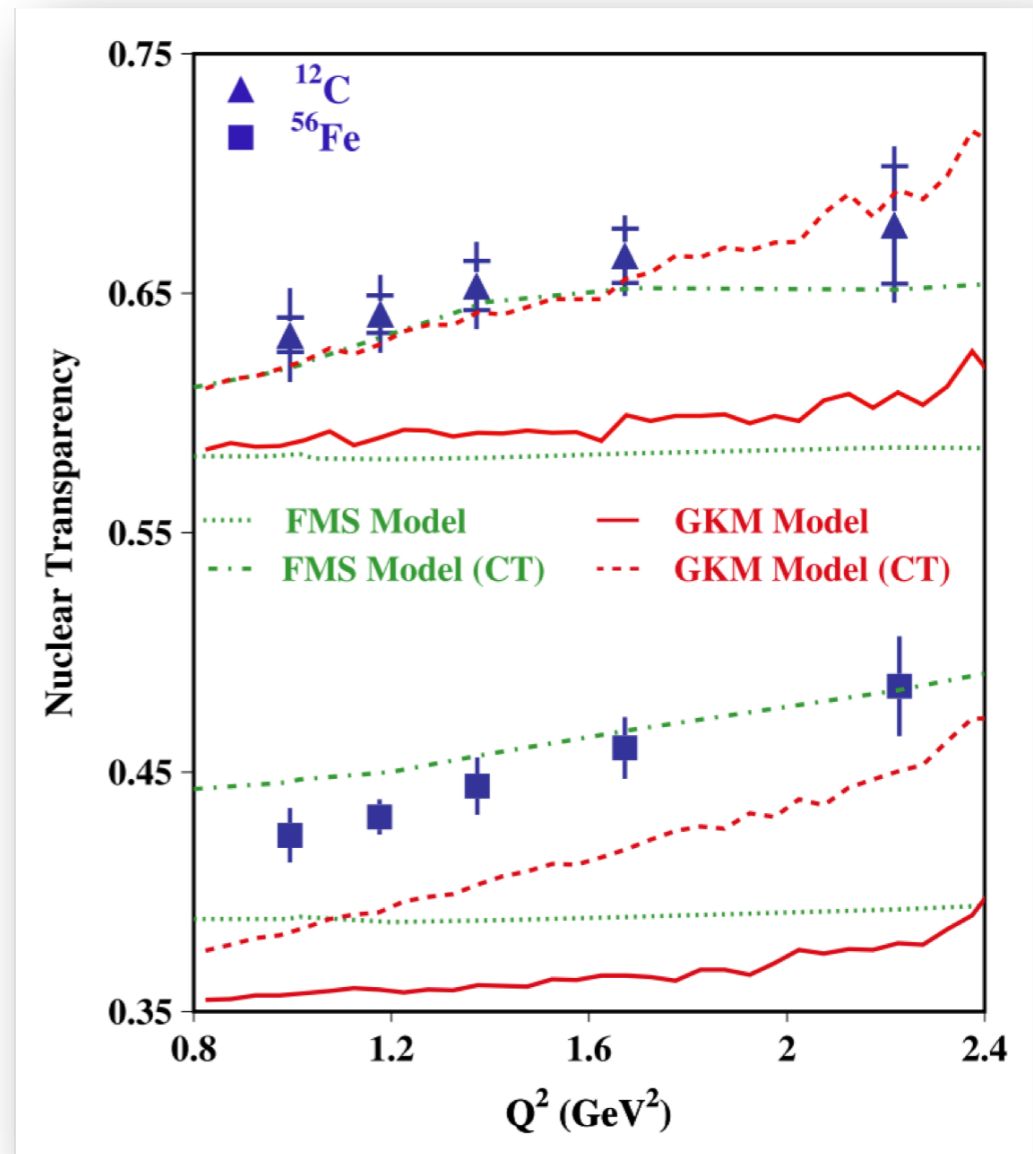
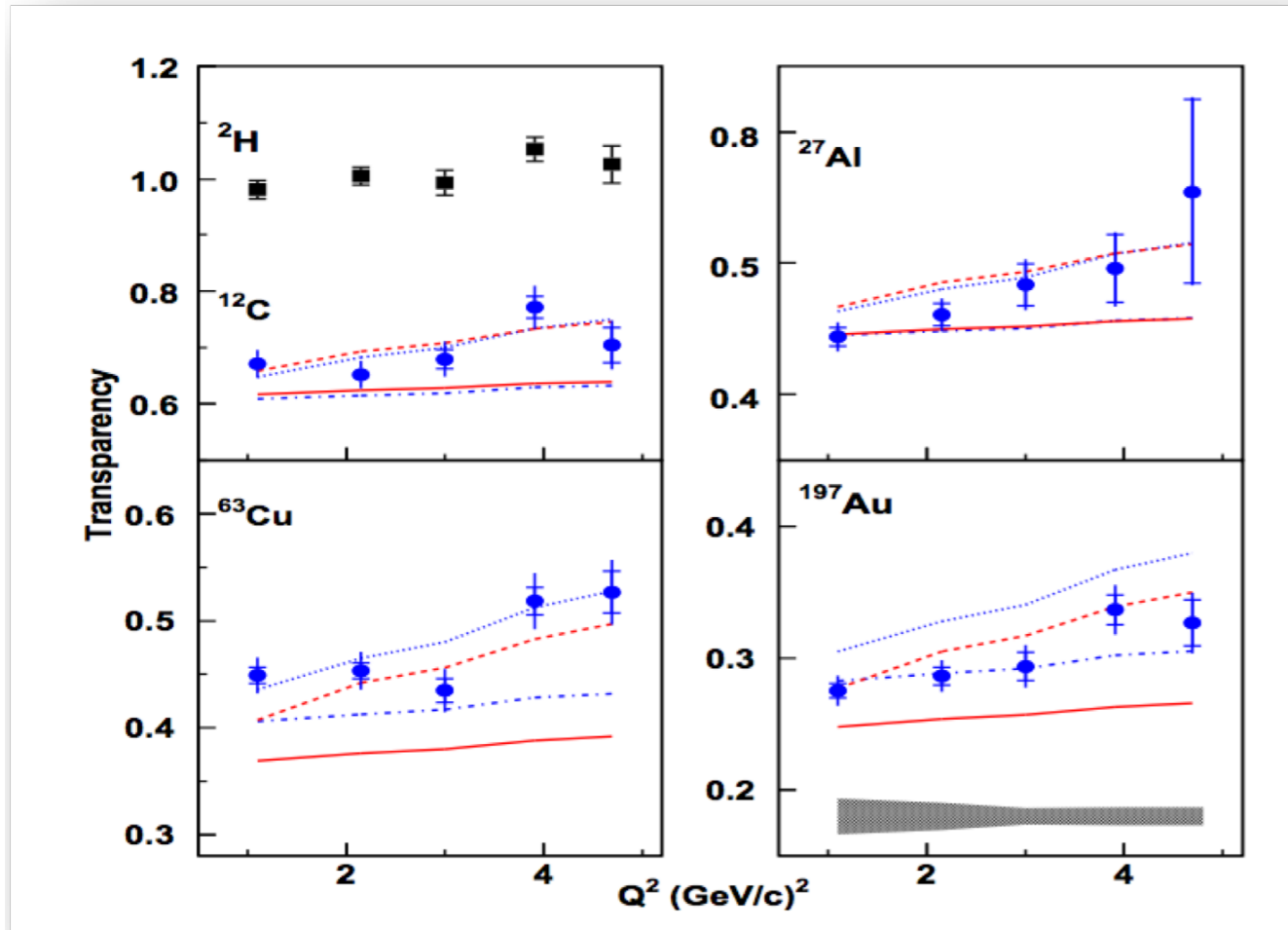


Proton has equal probability for  $\tau = 3$  and  $\tau = 4$

Hall C E01-107 pion electro-production

$$A(e, e' \pi^+)$$

$$A(e, e' \rho^0)$$



B.Glasie *et al.* PRL 99:242502 (2007)

X. Qian *et al.* PRC81:055209 (2010)

L. El Fassi *et al.* PLB 712,326 (2012)

$$\langle a_{\perp}^2(Q^2 = 4 \text{ GeV}^2) \rangle_{\tau=2} \simeq \langle a_{\perp}^2(Q^2 = 14 \text{ GeV}^2) \rangle_{\tau=3} \simeq \langle a_{\perp}^2(Q^2 = 22 \text{ GeV}^2) \rangle_{\tau=4} \simeq 0.24 \text{ fm}^2$$

5% increase for  $T_{\pi}$  in <sup>12</sup>C at  $Q^2 = 4 \text{ GeV}^2$  implies 5% increase for  $T_p$  at  $Q^2 = 18 \text{ GeV}^2$

- Transverse-impact size dependence on  $t = -Q^2$  from expectation value of the profile function  $\sigma(x)$

$$\begin{aligned}\langle \sigma(t) \rangle_\tau &= \frac{\int dx \sigma(x) \rho_\tau(x, t)}{\int dx \rho_\tau(x, t)} \\ &= \frac{1}{F_\tau(t)} \frac{d}{dt} F_\tau(t) = \frac{1}{4\lambda} [\psi(\tau - \alpha(t)) - \psi(1 - \alpha(t))]\end{aligned}$$

with  $\psi$  the digamma function

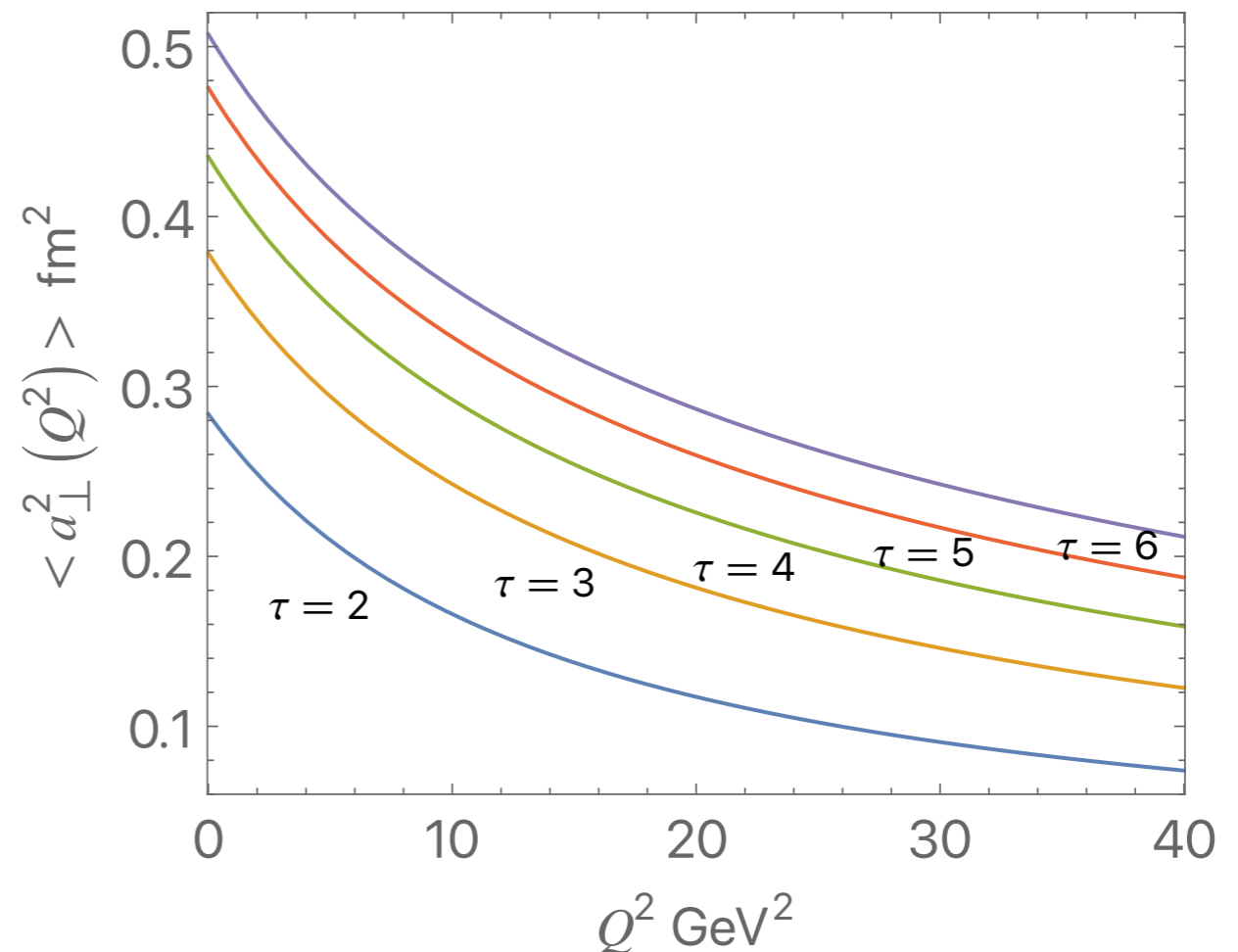
- For integer twist  $\tau = N$

$$\begin{aligned}\langle a_\perp^2(t) \rangle_\tau &\equiv 4 \langle \sigma(t) \rangle_\tau \\ &= \frac{1}{\lambda} \sum_{j=1}^{\tau-1} \frac{1}{j - \alpha(t)}\end{aligned}$$

- At large values  $t = -Q^2$

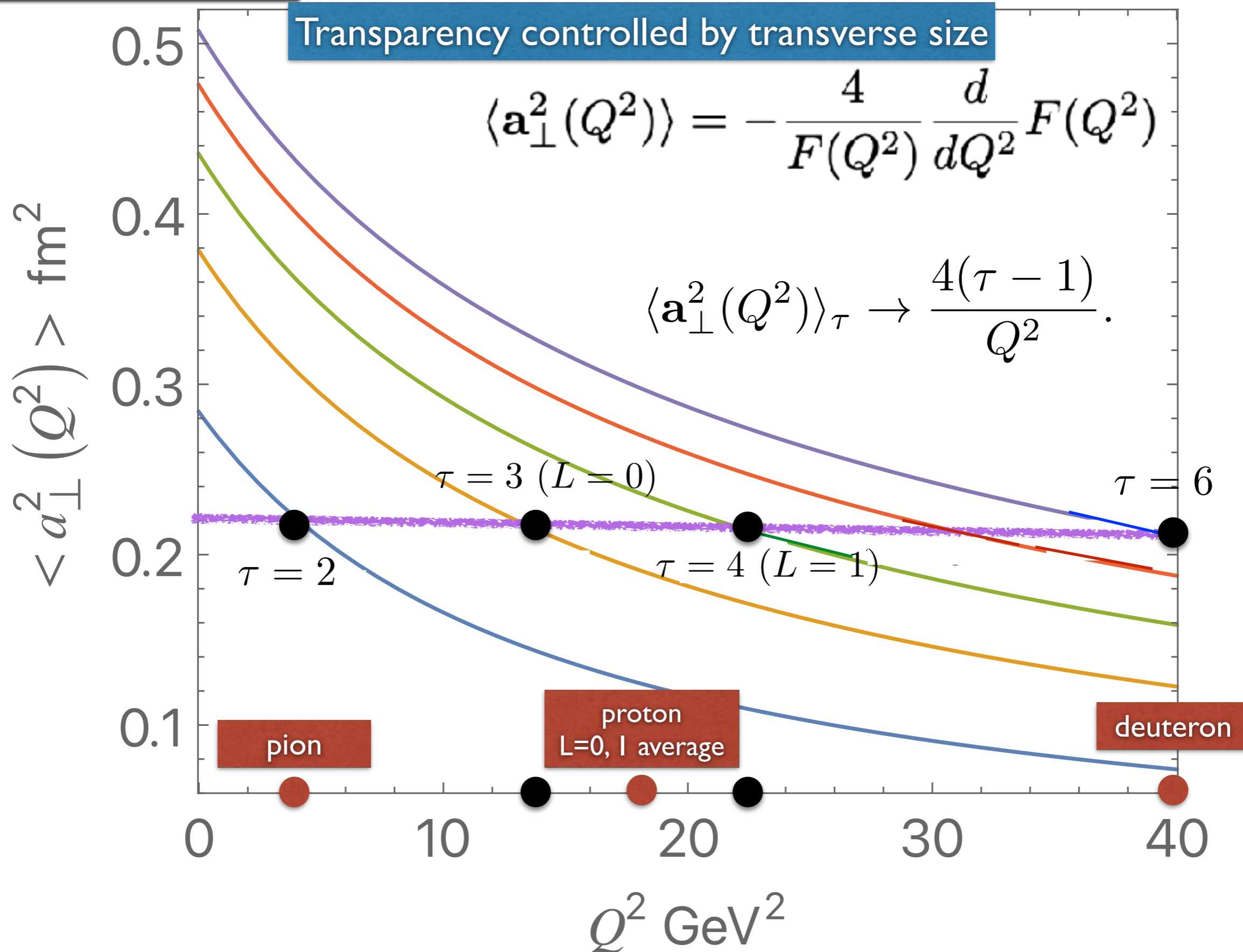
$$\langle a_\perp^2(Q^2) \rangle_\tau \rightarrow \frac{4(\tau - 1)}{Q^2}$$

- The  $Q^2$  required to contract all of the valence constituents of to a color-singlet domain of given transverse size, grows as the number of spectators and depends also on the properties of the quark current



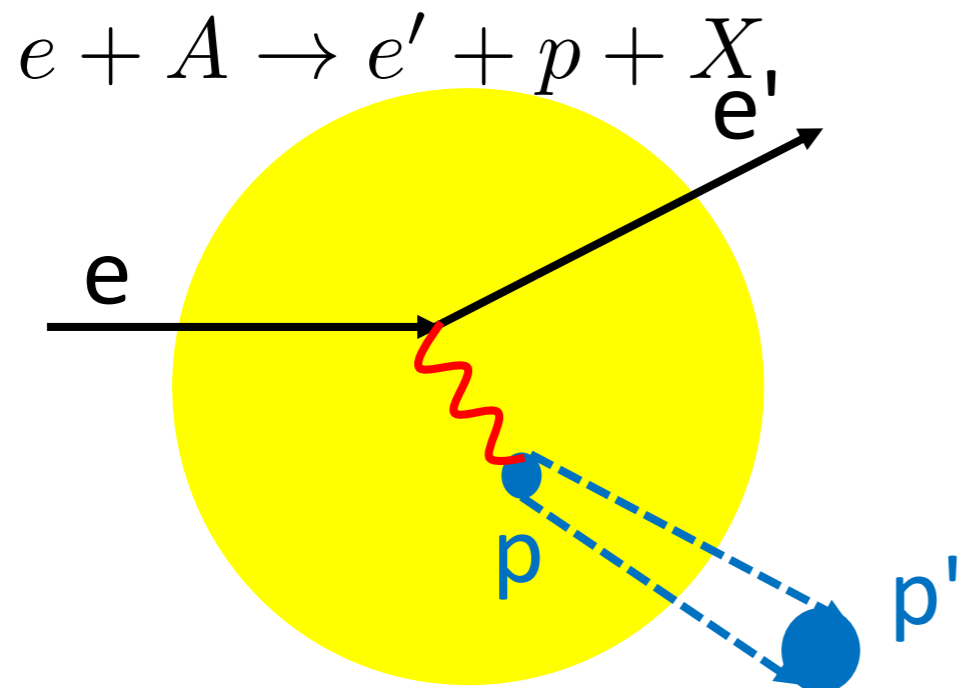
Transparency scale  $Q$   
increases with twist

Light-Front Holography

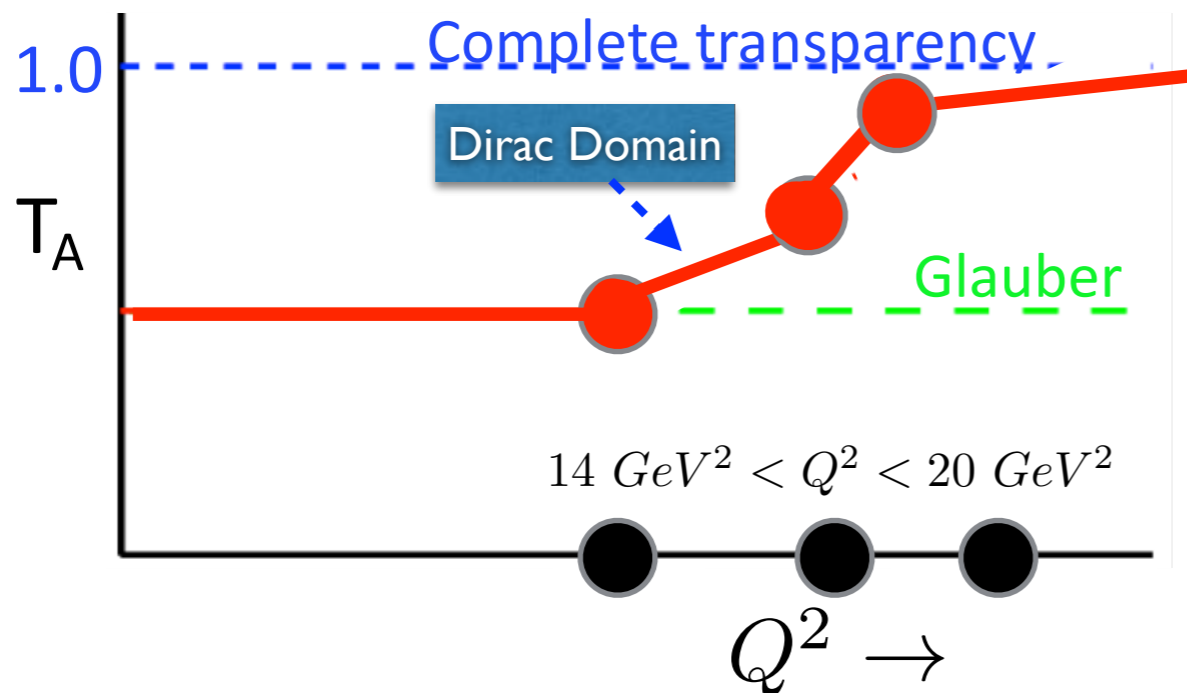


5% increase for  $T_{\pi}$  in  $^{12}\text{C}$  at  $Q^2 = 4 \text{ GeV}^2$  implies 5% increase for  $T_p$  at  $Q^2 = 18 \text{ GeV}^2$

# Color transparency fundamental prediction of QCD



- Not predicted by strongly interacting hadronic picture  $\rightarrow$  arises in picture of quark-gluon interactions
- QCD: color field of singlet objects vanishes as size is reduced
- Signature is a rise in nuclear transparency,  $T_A$ , as a function of the momentum transfer,  $Q^2$



$$T_A = \frac{\sigma_A}{A \sigma_N} \quad \begin{array}{l} \text{(nuclear cross section)} \\ \text{(free nucleon cross section)} \end{array}$$



# Two-Stage Color Transparency

$$14 \text{ GeV}^2 < Q^2 < 20 \text{ GeV}^2$$

If  $Q^2$  is in the intermediate range, then the twist-3 state will propagate through the nuclear medium with minimal absorption, and the protons which survive nuclear absorption will only have  $L = 0$  (twist-3).

The twist-4  $L = 1$  state which has a larger transverse size will be absorbed.

Thus 50% of the events in this range of  $Q^2$  will have full color transparency and 50% of the events will have zero color transparency ( $T = 0$ ).

√ The  $ep \rightarrow e'p'$  cross section will have the same angular and  $Q^2$  dependence as scattering of the electron on an unphysical proton which has no Pauli form factor.

$$Q^2 > 20 \text{ GeV}^2$$

However, if the momentum transfer is increased to  $Q^2 > 20 \text{ GeV}^2$ , all events will have full color transparency, and the  $ep \rightarrow e'p'$  cross section will have the same angular and  $Q^2$  dependence as scattering of the electron on a physical proton eigenstate, with both Dirac and Pauli form factor components.

# *Color Transparency and Light-Front Holography*

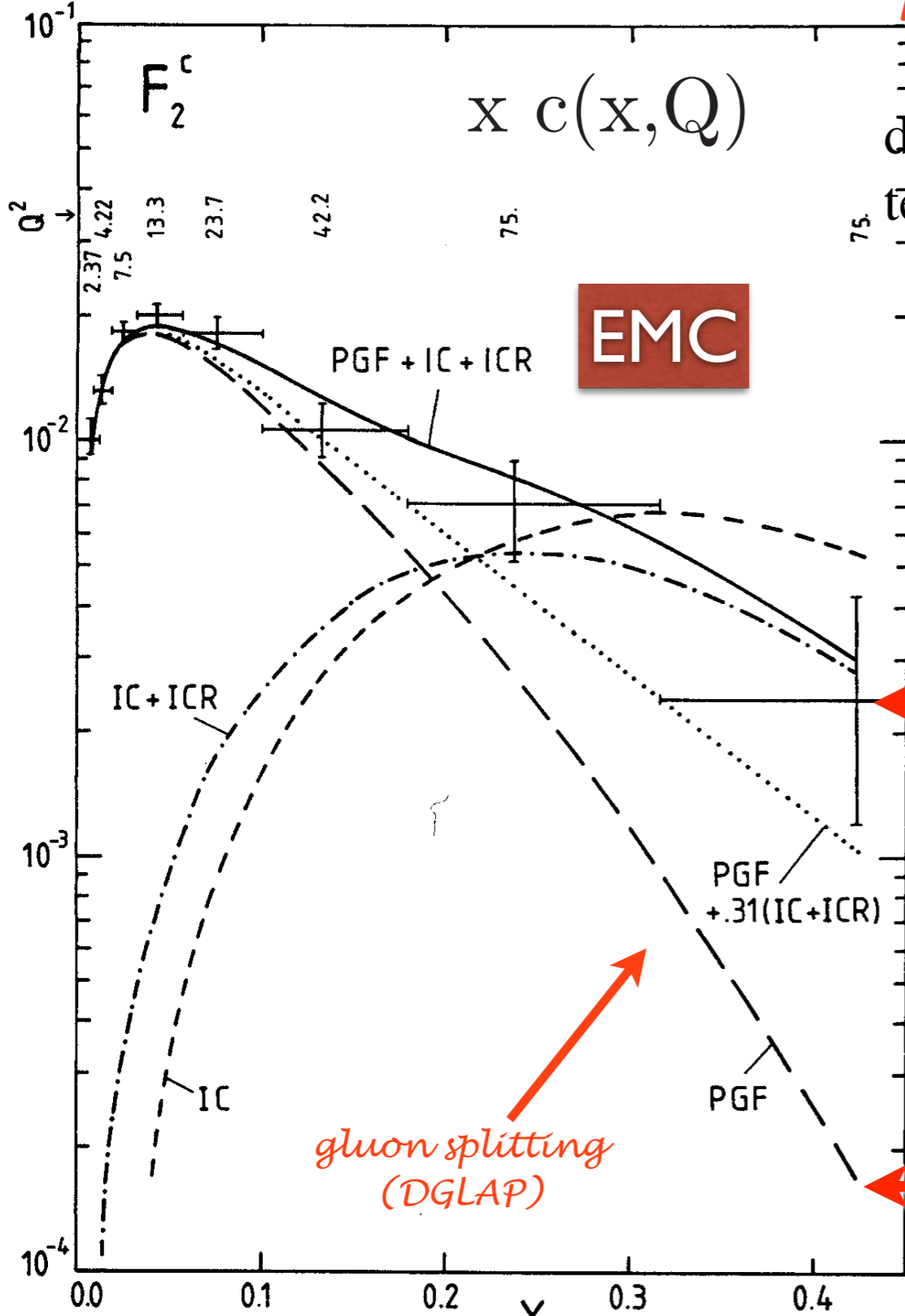
- Essential prediction of QCD
- LF Holography: Spectroscopy, dynamics, structure
- Transverse size predicted by LF Holography as a function of Q
- Q scale for CT increases with twist, number of constituents
- Two-Stage Proton Transparency: Equal probability L=0,1
- No contradiction with present experiments

$Q_0^2(p) \simeq 18 \text{ GeV}^2$  vs.  $Q_0^2(\pi) \simeq 4 \text{ GeV}^2$  for onset of color transparency in  $^{12}\text{C}$

$$Q_0^2(d) \simeq 40 \text{ GeV}^2$$

*Measurement of Charm Structure Function!*

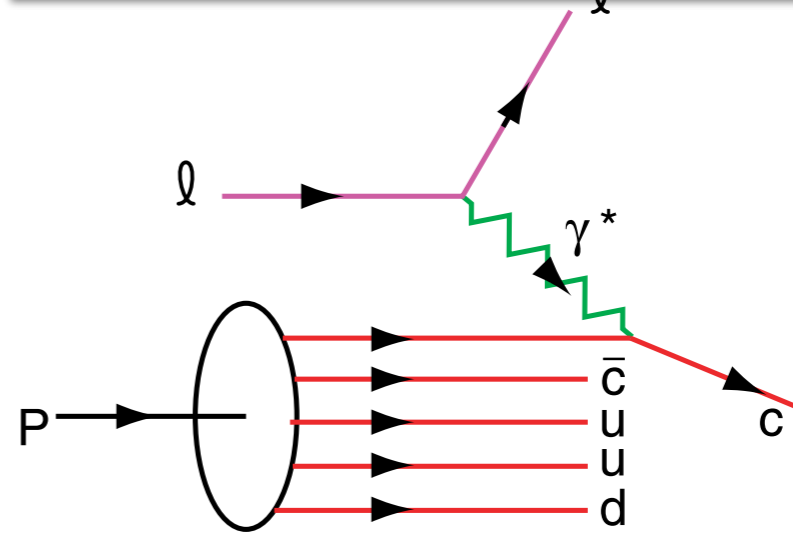
J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-GeV Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).



**Evidence for Intrinsic Charm**

$$\langle x_{c\bar{c}} \rangle_p \simeq 1\%$$

New Analysis:  
 R.D. Ball, et al. [NNPDF Collaboration],  
 "A Determination of the Charm Content  
 of the Proton,"  
 arXiv:1605.06515 [hep-ph].



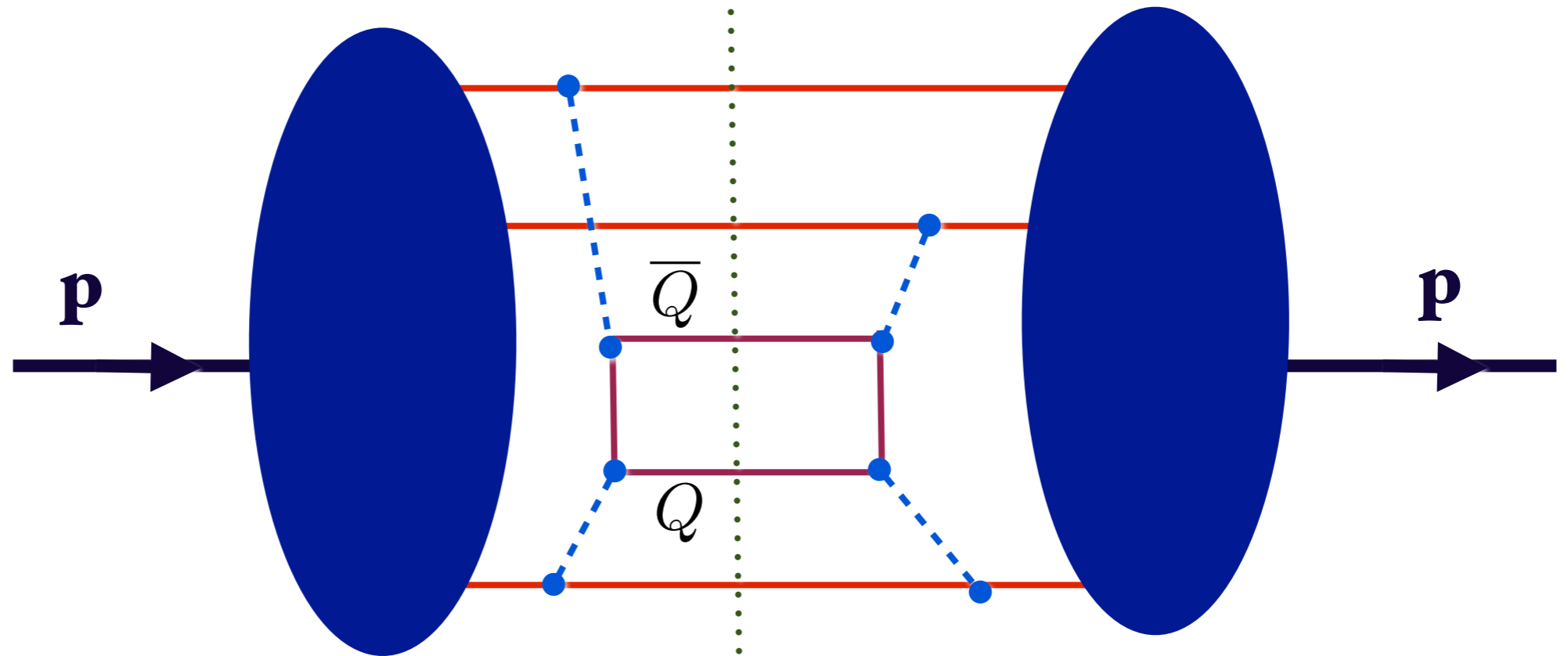
**DGLAP / Photon-Gluon Fusion: factor of 30 too small**

*Two Components (separate evolution):*

$$c(x, Q^2) = c(x, Q^2)_{\text{extrinsic}} + c(x, Q^2)_{\text{intrinsic}}$$

*Proton Self Energy*  
*Intrinsic Heavy Quarks*

*Fixed LF time*



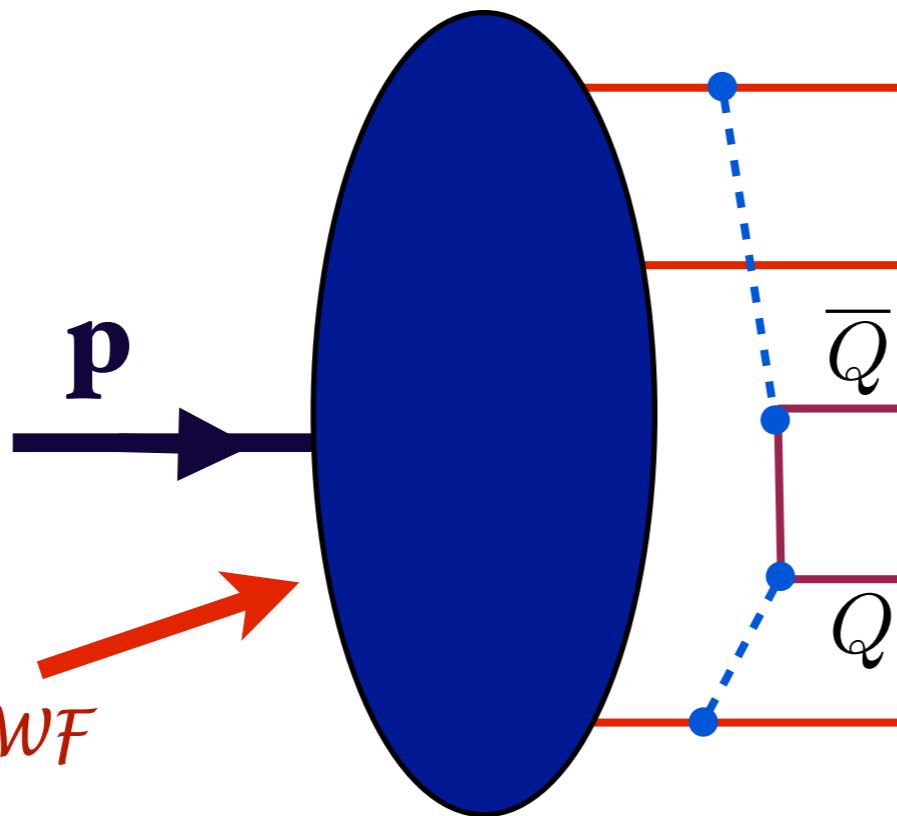
Probability (QED)  $\propto \frac{1}{M_\ell^4}$

Probability (QCD)  $\propto \frac{1}{M_Q^2}$

**Rigorous OPE Analysis**

**Collins, Ellis, Gunion, Mueller, sjb  
M. Polyakov, et al.**

*Proton 5-quark Fock State:  
Intrinsic Heavy Quarks*



*Use AdS/QCD LFWF*

$$x_Q \propto (m_Q^2 + k_{\perp}^2)^{1/2}$$

$$\text{Probability (QED)} \propto \frac{1}{M_{\ell}^4}$$

$g \rightarrow Q\bar{Q}$  at low  $x$ : High  $\mathcal{M}^2$

*QCD predicts  
Intrinsic  
Heavy Quarks  
at high  $x$ !*

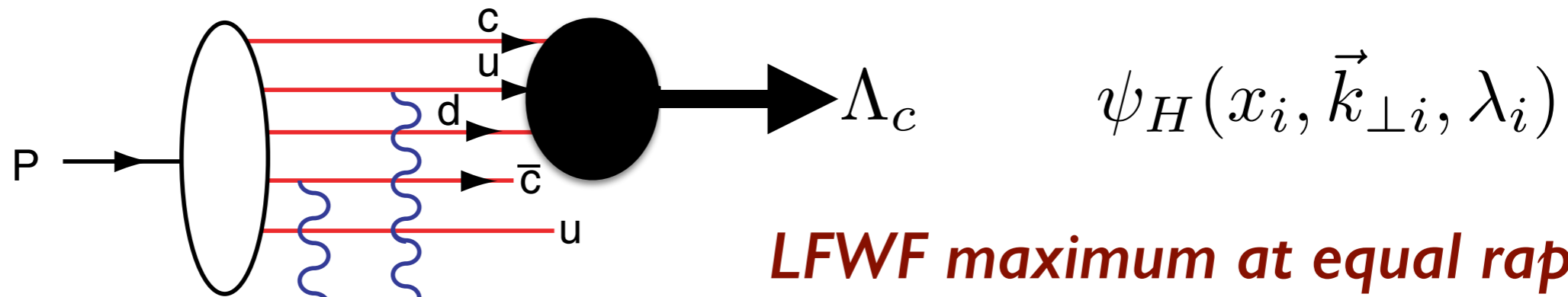
**Minimal off-shellness!**

$$\text{Probability (QCD)} \propto \frac{1}{M_Q^2}$$

**Collins, Ellis, Gunion, Mueller, sjb  
Polyakov, et al.**

# Coalescence of comovers produces high $x_F$ heavy hadrons

High  $x_F$  hadrons combine most of the comovers, fewest spectators



*LFWF maximum at equal rapidity*

*maximum at minimal invariant mass*

**X**  $\rightarrow$  *Asymmetries of leading hadrons*

**Spectator counting rules**

$$\frac{dN}{dx_F} \propto (1 - x_F)^{2n_{spect} - 1}$$

Coalescence of Comoving Charm and Valence Quarks  
Produce  $J/\psi$ ,  $\Lambda_c$  and other Charm Hadrons at High  $x_F$

Vogt, sjb

Stan Brodsky

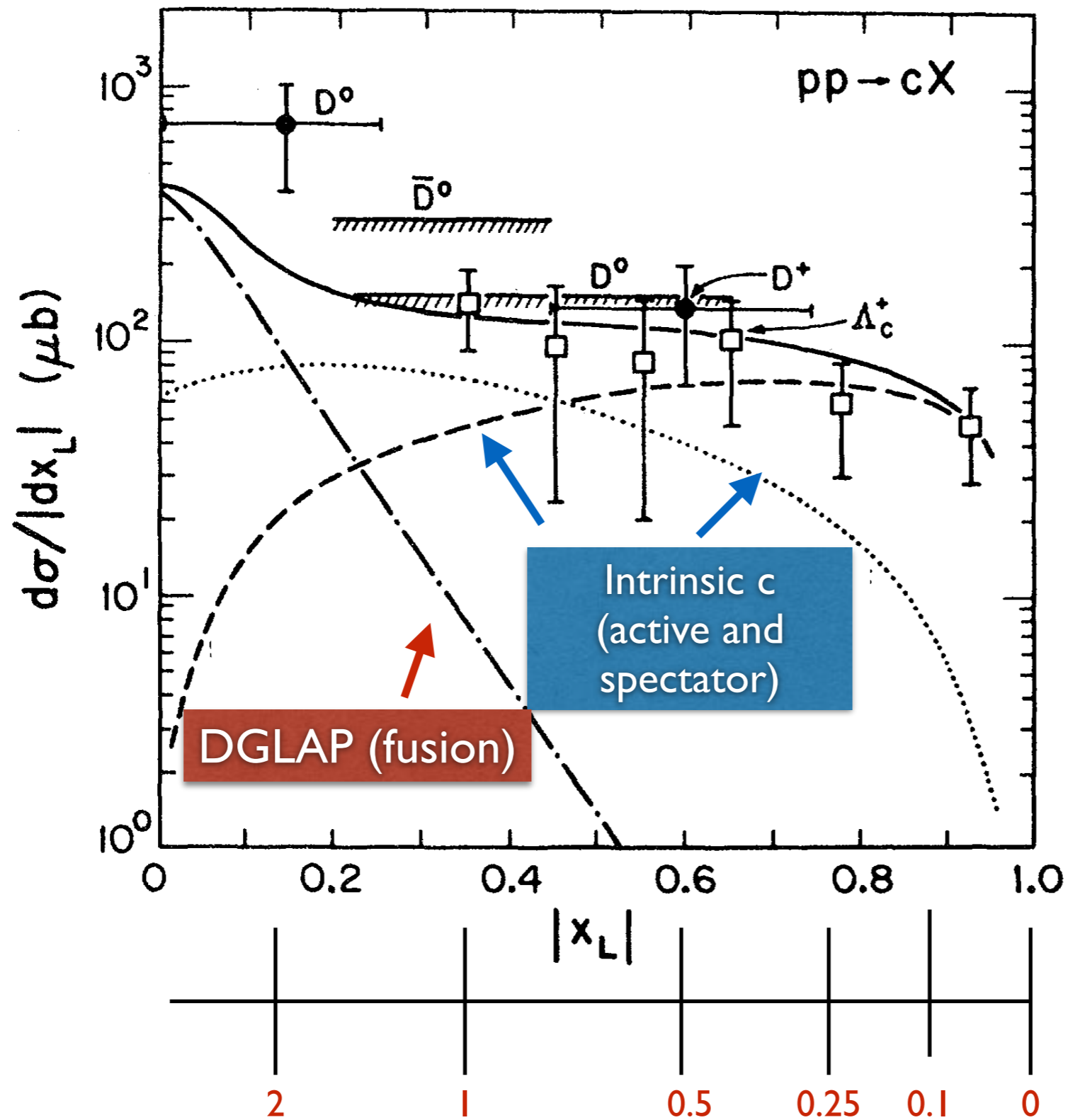


Supersymmetric Features of Hadron Physics  
from Superconformal Algebra  
and Light-Front Holography



19 April 2021

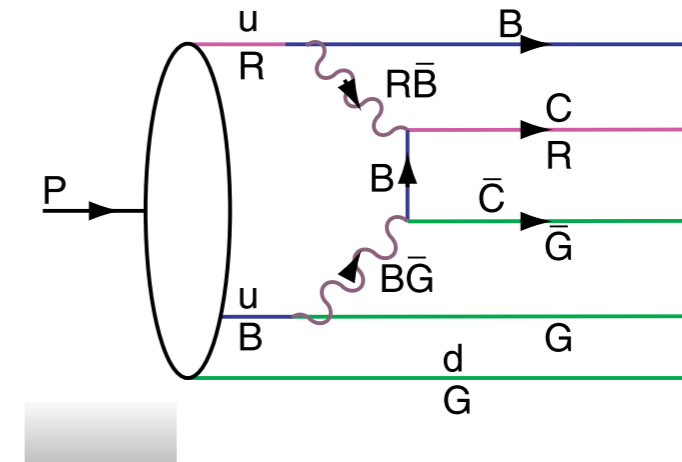




$$\Delta y = \log x$$

# Intrinsic Heavy-Quark Fock States

- Rigorous prediction of QCD, OPE
- Color-Octet Color-Octet Fock State!



- Probability  $P_{Q\bar{Q}} \propto \frac{1}{M_Q^2}$   $P_{QQ\bar{Q}\bar{Q}} \sim \alpha_s^2 P_{Q\bar{Q}}$   $P_{c\bar{c}/p} \simeq 1\%$

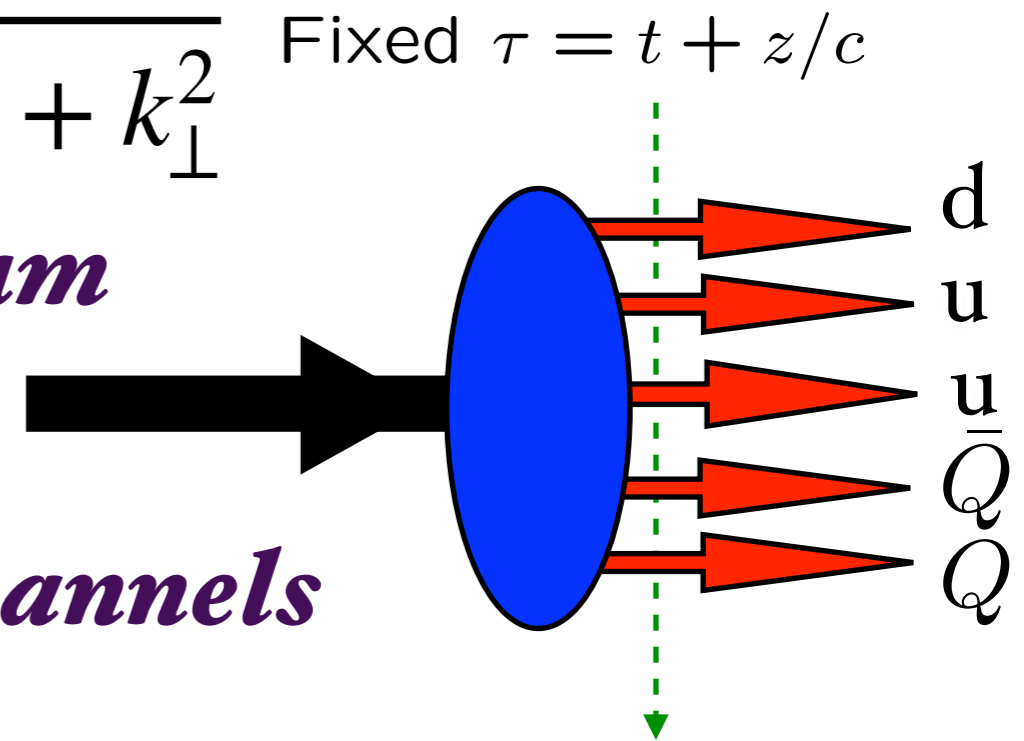
- Large Effect at high x
- Greatly increases kinematics of colliders such as Higgs production at high  $x_F$  (Kopeliovich, Schmidt, Soffer, Goldhaber, sjb)
- Severely underestimated in conventional parameterizations of heavy quark distributions (Pumplin, Tung)
- Many empirical tests (Gardener, Karliner, ..)

Review: G. Lykasov, et al

# Properties of Non-Perturbative Five-Quark Fock-State

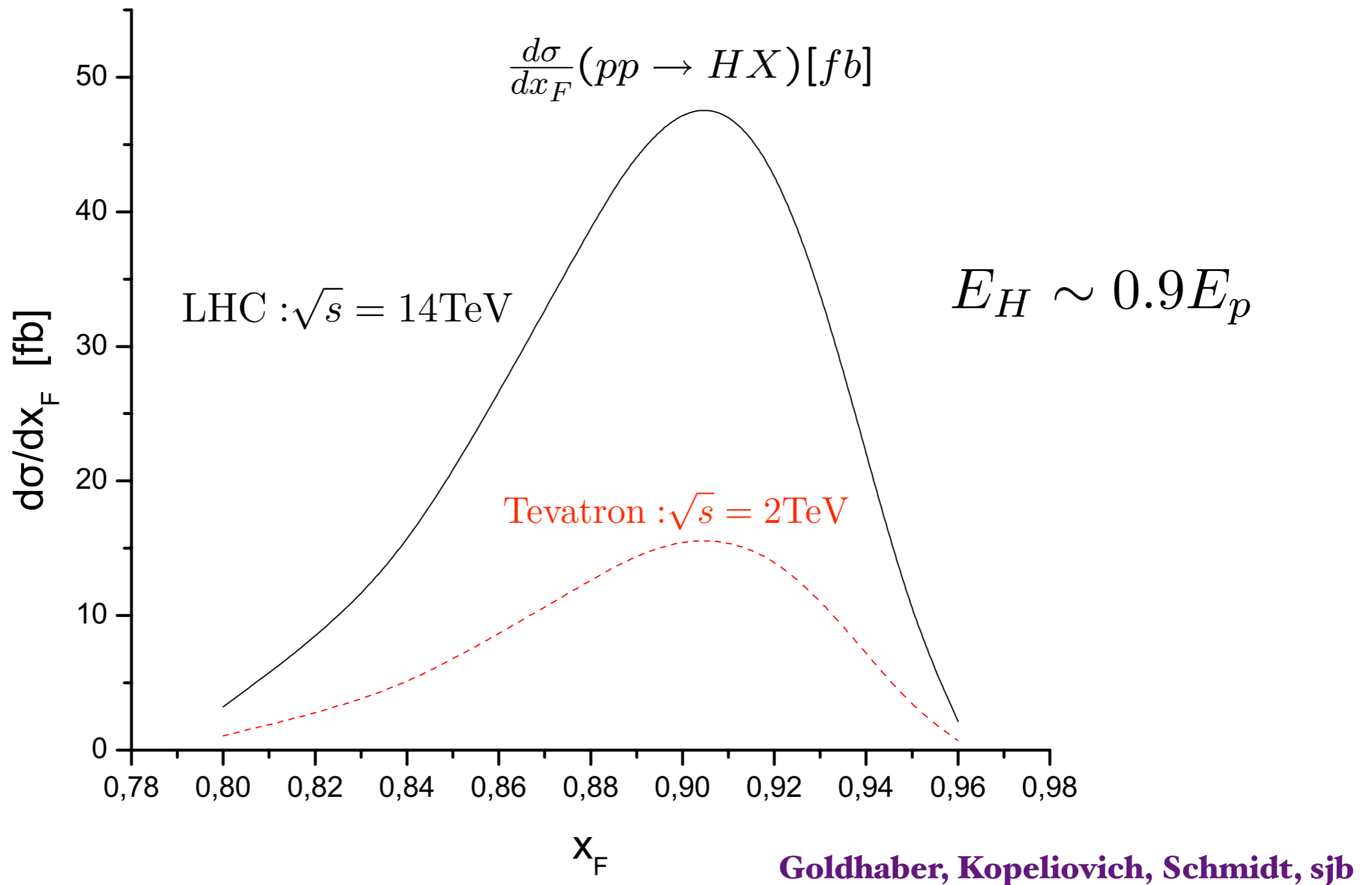
- **Dominant configuration: minimum off-shell, same rapidity**
- **Heavy quarks have most of the LF momentum**

$$\langle x_Q \rangle \propto \sqrt{m_Q^2 + k_{\perp}^2}$$
- **Correlated with proton quantum numbers**
- **Duality with meson-baryon channels**
- **Strangeness, charm asymmetry at  $x > 0.1$**



$$s_p(x) \neq \bar{s}_p(x) \quad c_p(x) \neq \bar{c}_p(x)$$

# Intrinsic Heavy Quark Contribution to Inclusive Higgs Production



Measure  $H \rightarrow ZZ^* \rightarrow \mu^+ \mu^- \mu^+ \mu^-$ .

# Constraints on charm-anticharm asymmetry in the nucleon from lattice QCD

Raza Sabbir Sufian<sup>a</sup>, Tianbo Liu<sup>a</sup>, Andrei Alexandru<sup>b,c</sup>, Stanley J. Brodsky<sup>d</sup>, Guy F. de Téramond<sup>e</sup>,  
Hans Günter Dosch<sup>f</sup>, Terrence Draper<sup>g</sup>, Keh-Fei Liu<sup>g</sup>, Yi-Bo Yang<sup>h</sup>

<sup>a</sup>*Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA*

<sup>b</sup>*Department of Physics, The George Washington University, Washington, DC 20052, USA*

<sup>c</sup>*Department of Physics, University of Maryland, College Park, MD 20742, USA*

<sup>d</sup>*SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94309, USA*

<sup>e</sup>*Laboratorio de Física Teórica y Computacional, Universidad de Costa Rica, 11501 San José, Costa Rica*

<sup>f</sup>*Institut für Theoretische Physik der Universität, D-69120 Heidelberg, Germany*

<sup>g</sup>*Department of Physics and Astronomy, University of Kentucky, Lexington, Kentucky 40506, USA*

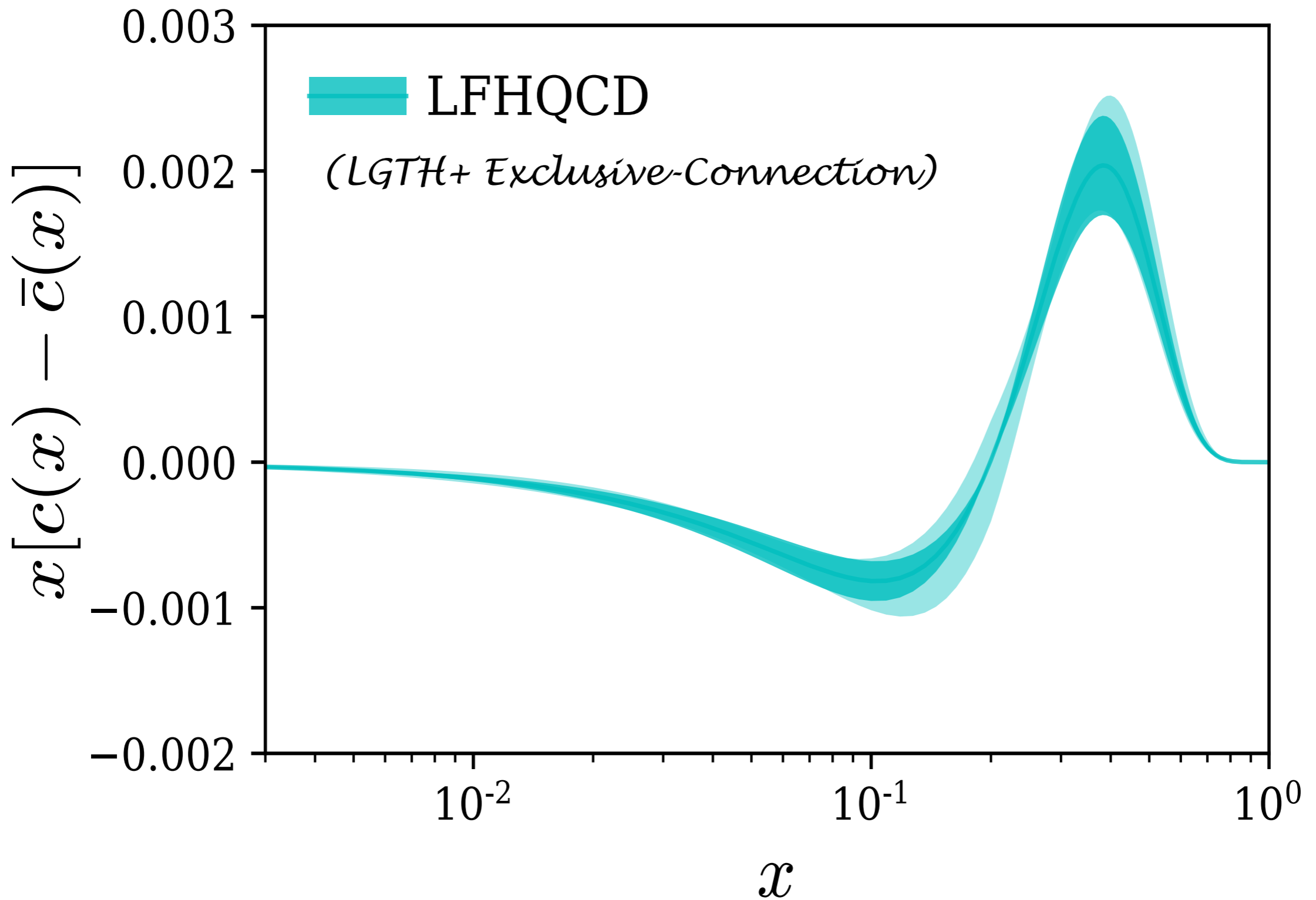
<sup>h</sup>*CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China*

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## Abstract

We present the first lattice QCD calculation of the charm quark contribution to the nucleon electromagnetic form factors  $G_{E,M}^c(Q^2)$  in the momentum transfer range  $0 \leq Q^2 \leq 1.4 \text{ GeV}^2$ . The quark mass dependence, finite lattice spacing and volume corrections are taken into account simultaneously based on the calculation on three gauge ensembles including one at the physical pion mass. The nonzero value of the charm magnetic moment  $\mu_M^c = -0.00127(38)_{\text{stat}}(5)_{\text{sys}}$ , as well as the Pauli form factor, reflects a nontrivial role of the charm sea in the nucleon spin structure. The nonzero  $G_E^c(Q^2)$  indicates the existence of a nonvanishing asymmetric charm-anticharm sea in the nucleon. Performing a non-perturbative analysis based on holographic QCD and the generalized Veneziano model, we study the constraints on the  $[c(x) - \bar{c}(x)]$  distribution from the lattice QCD results presented here. Our results provide complementary information and motivation for more detailed studies of physical observables that are sensitive to intrinsic charm and for future global analyses of parton distributions including asymmetric charm-anticharm distribution.

*Keywords:* Intrinsic charm, Form factor, Parton distributions, Lattice QCD, Light-front holographic QCD, JLAB-THY-20-3155, SLAC-PUB-17515



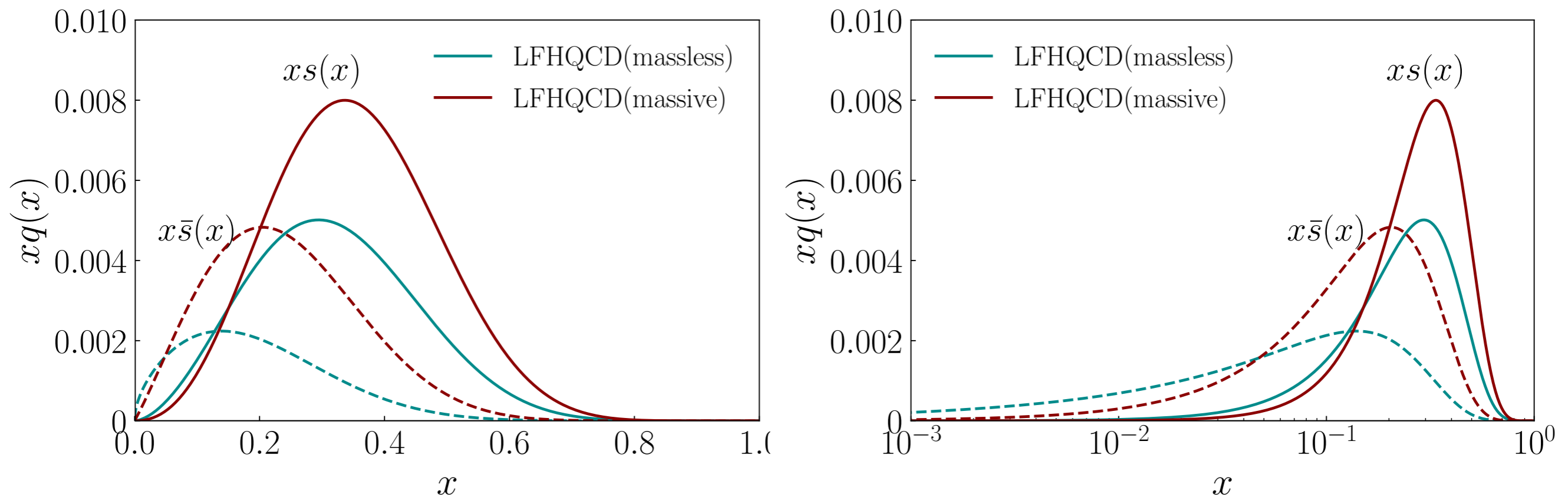
The distribution function  $x[c(x) - \bar{c}(x)]$  obtained from the LFHQCD formalism using the lattice QCD input of charm electromagnetic form factors  $G_{E,M}^c(Q^2)$ . The outer cyan band indicates an estimate of systematic uncertainty in the  $x[c(x) - \bar{c}(x)]$  distribution obtained from a variation of the hadron scale  $\kappa_c$  by 5%.



# Strange and Antistrange Distributions

Input: nonzero lattice axial form factor

Duality with  $|K\Lambda\rangle$  meson-nucleon fluctuations



Phys. Rev. D 98, 114004 (2018).

R. S. Sufian, T. Liu, de Teramond, Dosch, Deur, Islam, Ma, *sjb*

# Challenge: Compute Hadron Structure, Spectroscopy, and Dynamics from QCD!

- **Color Confinement**
- **Origin of the QCD Mass Scale**
- **Meson and Baryon Spectroscopy**
- **Exotic States: Tetraquarks, Pentaquarks, Gluonium,**
- **Universal Regge Slopes:  $n$ ,  $L$ , Mesons and Baryons**
- **Almost Massless Pion: GMOR Chiral Symmetry Breaking**  
$$M_\pi^2 f_\pi^2 = -\frac{1}{2}(m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle + \mathcal{O}((m_u + m_d)^2)$$
- **QCD Coupling at all Scales  $\alpha_s(Q^2)$**
- **Eliminate Scale Uncertainties and Scheme Dependence**

$$\mathcal{L}_{QCD} \rightarrow \psi_n^H(x_i, \vec{k}_{\perp i}, \lambda_i) \quad \text{Valence and Higher Fock States}$$

# Light-Front Holography: First Approximation to QCD

- **Color Confinement, Analytic form of confinement potential**
- **Retains underlying conformal properties of QCD despite mass scale (DeAlfaro-Fubini-Furlan Principle)**
- **Massless quark-antiquark pion bound state in chiral limit, GMOR-like relation**
- **QCD coupling at all scales**
- **Nonperturbative mass scale**
- **Hadron Spectroscopy-Regge Trajectories for mesons, baryons with universal slopes in  $n, L$**
- **Supersymmetric 4-Plet: Meson-Baryon-Tetraquark Symmetry**
- **Light-Front Wavefunctions**
- **Form Factors, Quark and Gluon Structure Functions**
- **New Hadronic Observables; gluonic radii and gluonic momentum fraction**
- **OPE: Constituent Counting Rules**
- **Hadronization at the Amplitude Level: Many Phenomenological Tests**
- **Systematically improvable: Basis LF Quantization (BLFQ)**

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*Bled Workshop*

*Supersymmetric Features of Hadron Physics  
from Superconformal Algebra  
and Light-Front Holography*

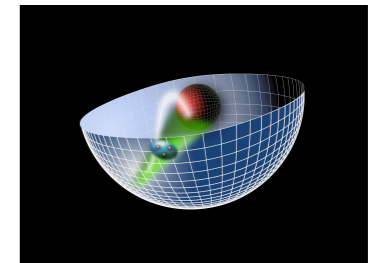


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# LFHQCD: Underlying Principles

- **Poincarè Invariance: Independent of the observer's Lorentz frame: Quantization at Fixed Light-Front Time  $\tau$**
- **Causality: Information within causal horizon: Light-Front**
- **Light-Front Holography:  $AdS_5 = LF (3+1)$**

$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_{\perp}^2 x(1-x)$$



- **Introduce Mass Scale  $\kappa$  while retaining the Conformal Invariance of the Action (dAFF)**
- **Unique Dilaton in  $AdS_5$ :  $e^{+\kappa^2 z^2}$**
- **Unique color-confining LF Potential  $U(\zeta^2) = \kappa^4 \zeta^2$**
- **Superconformal Algebra: Mass Degenerate 4-Plet:**

Meson  $q\bar{q} \leftrightarrow$  Baryon  $q[qq] \leftrightarrow$  Tetraquark  $[qq][\bar{q}\bar{q}]$

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**Supersymmetric Features of Hadron Physics**  
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# *Invariance Principles of Quantum Field Theory*

- **Polncarè Invariance:** Physical predictions must be independent of the observer's Lorentz frame: *Front Form*
- **Causality:** Information within causal horizon: *Front Form*
- **Gauge Invariance:** Physical predictions of gauge theories must be independent of the choice of gauge
- **Scheme-Independence:** Physical predictions of renormalizable theories must be independent of the choice of the renormalization scheme — *Principle of Maximum Conformality (PMC)*
- **Mass-Scale Invariance:**  
*Conformal Invariance of the Action (DAFF)*

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7 July 2021



## **Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD**

Matin Mojaza\*

*CP3-Origins, Danish Institute for Advanced Studies, University of Southern Denmark, DK-5230 Odense, Denmark  
and SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA*

Stanley J. Brodsky†

*SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA*

Xing-Gang Wu‡

*Department of Physics, Chongqing University, Chongqing 401331, People's Republic of China  
(Received 13 January 2013; published 10 May 2013)*

We introduce a generalization of the conventional renormalization schemes used in dimensional regularization, which illuminates the renormalization scheme and scale ambiguities of perturbative QCD predictions, exposes the general pattern of nonconformal  $\{\beta_i\}$  terms, and reveals a special degeneracy of the terms in the perturbative coefficients. It allows us to systematically determine the argument of the running coupling order by order in perturbative QCD in a form which can be readily automatized. The new method satisfies all of the principles of the renormalization group and eliminates an unnecessary source of systematic error.



# Set multiple renormalization scales -- Lensing, DGLAP, ERBL Evolution ...

Choose renormalization scheme; e.g.  $\alpha_s^R(\mu_R^{\text{init}})$

Choose  $\mu_R^{\text{init}}$ ; arbitrary initial renormalization scale

Identify  $\{\beta_i^R\}$  – terms using  $n_f$  – terms  
through the PMC – BLM correspondence principle

Shift scale of  $\alpha_s$  to  $\mu_R^{\text{PMC}}$  to eliminate  $\{\beta_i^R\}$  – terms

Conformal Series

Result is independent of  $\mu_R^{\text{init}}$  and scheme at fixed order

## PMC/BLM

**No renormalization scale ambiguity!**

*Result is independent of  
Renormalization scheme  
and initial scale!*

**QED Scale Setting at  $N_c=0$**

**Eliminates unnecessary  
systematic uncertainty**

**Scale fixed at each order**

**$\delta$ -Scheme automatically  
identifies  $\beta$ -terms!**

## Principle of Maximum Conformality

*Xing-Gang Wu, Martin Mojaza  
Leonardo di Giustino, Sfb*

**Stan Brodsky**



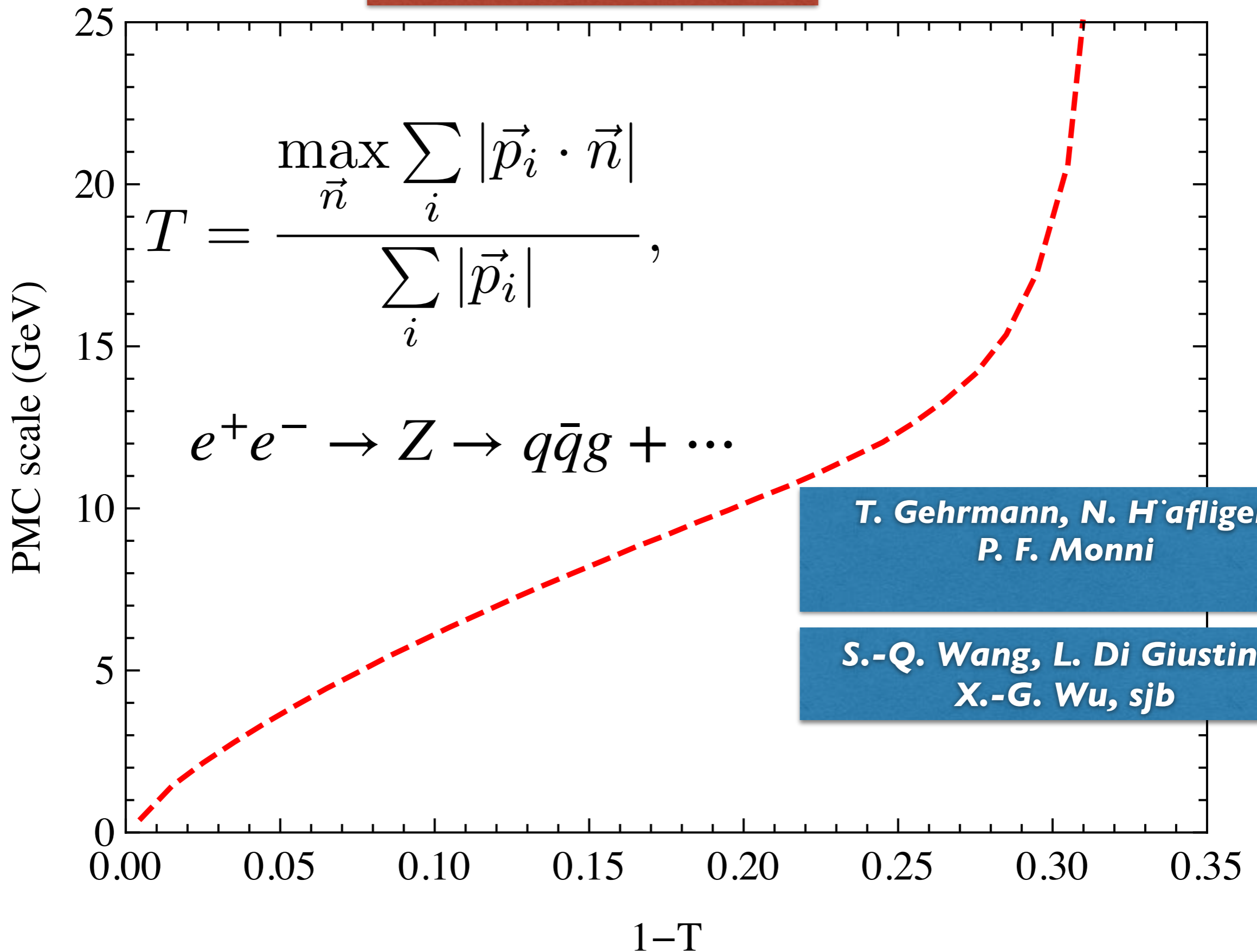
**Supersymmetric Features of Hadron Physics  
from Superconformal Algebra  
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28 May 2020

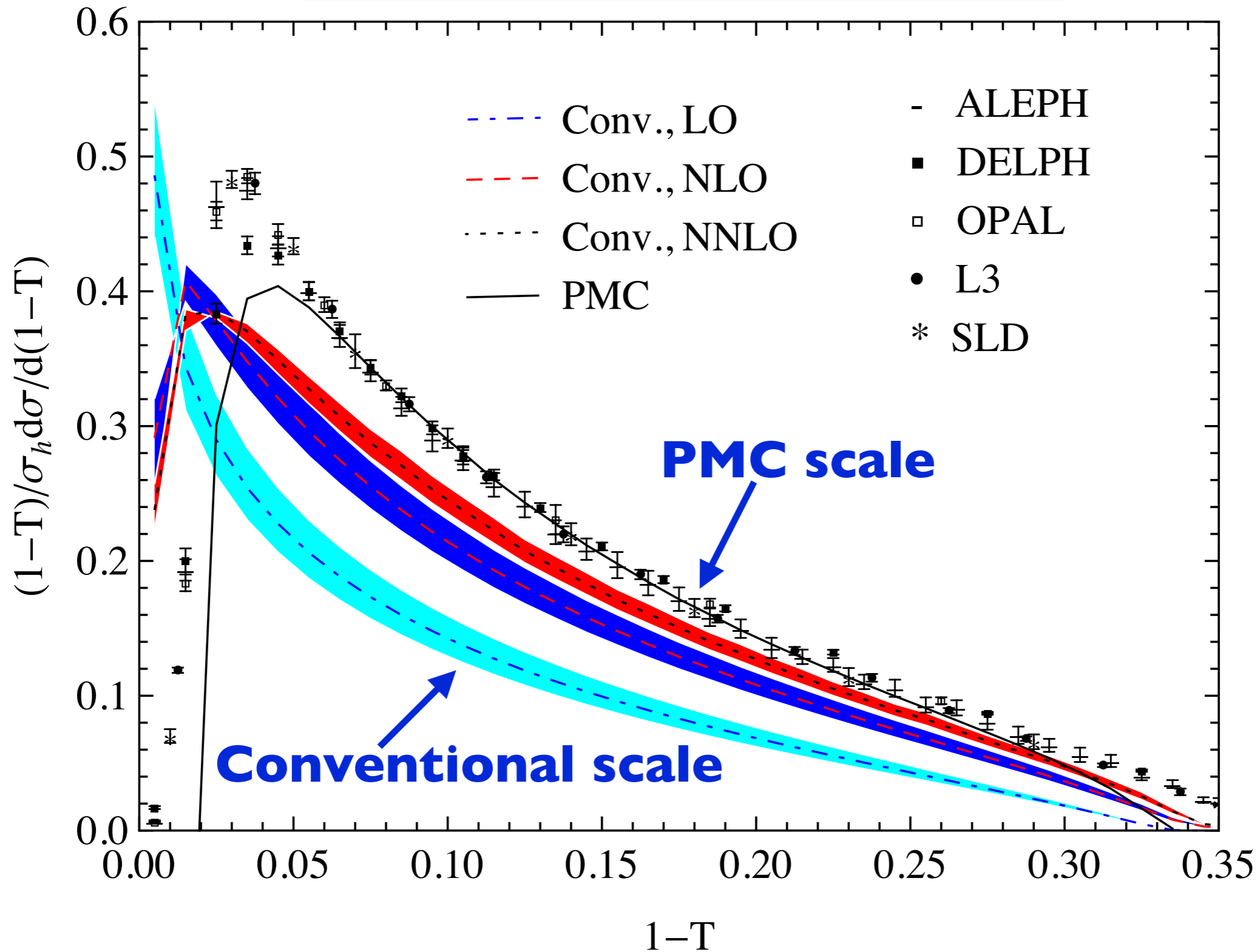
# Renormalization scale depends on the thrust

**Not constant !**



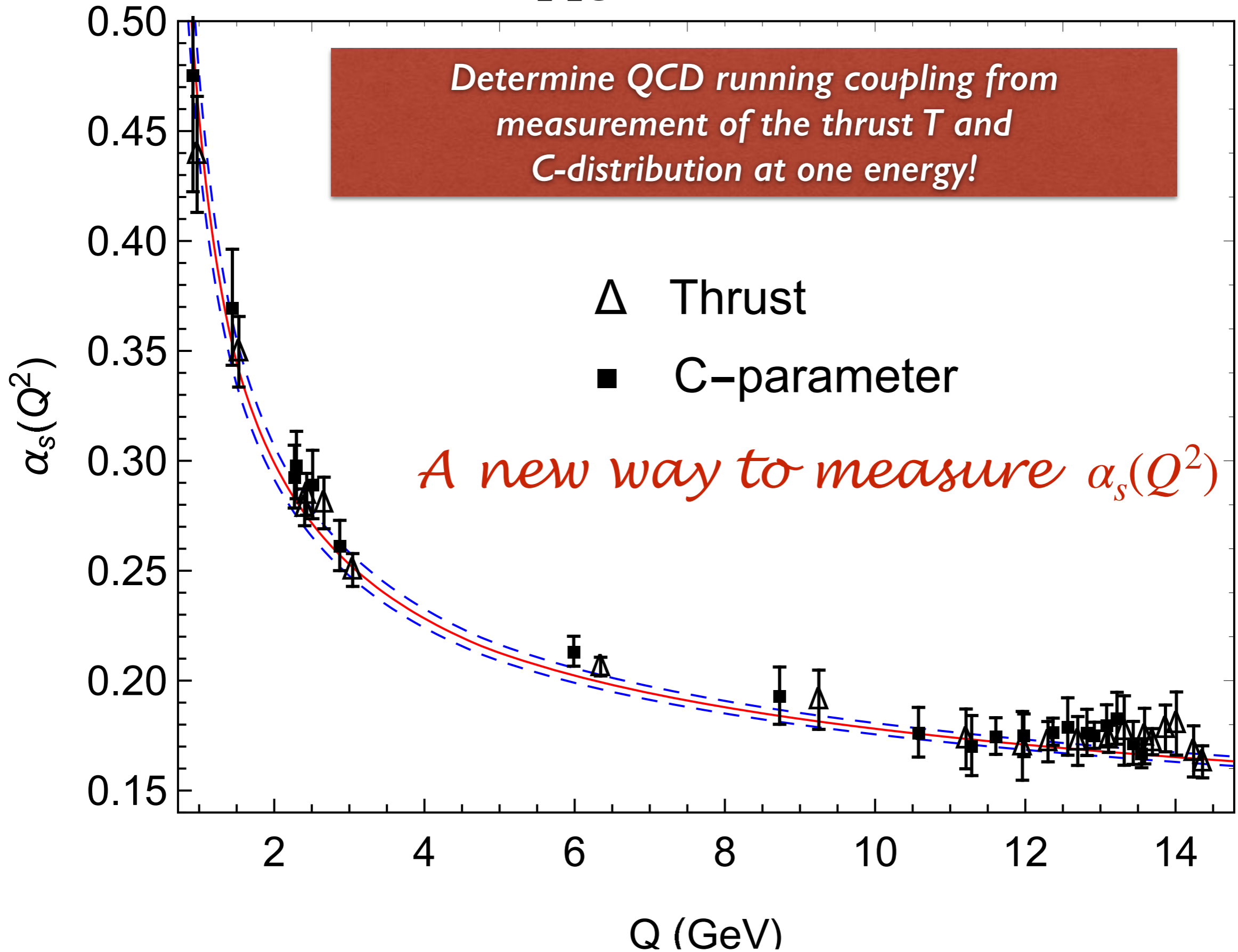
T. Gehrmann, N. H'afliker, P. F. Monni

S.-Q. Wang, L. Di Giustino, X.-G. Wu, sjb



**Principle of Maximum Conformality (PMC)**

$e^+e^- \rightarrow Z^0 \rightarrow q\bar{q}g + \dots$   $\alpha_s(Q^2)$  in  $\overline{MS}$  scheme

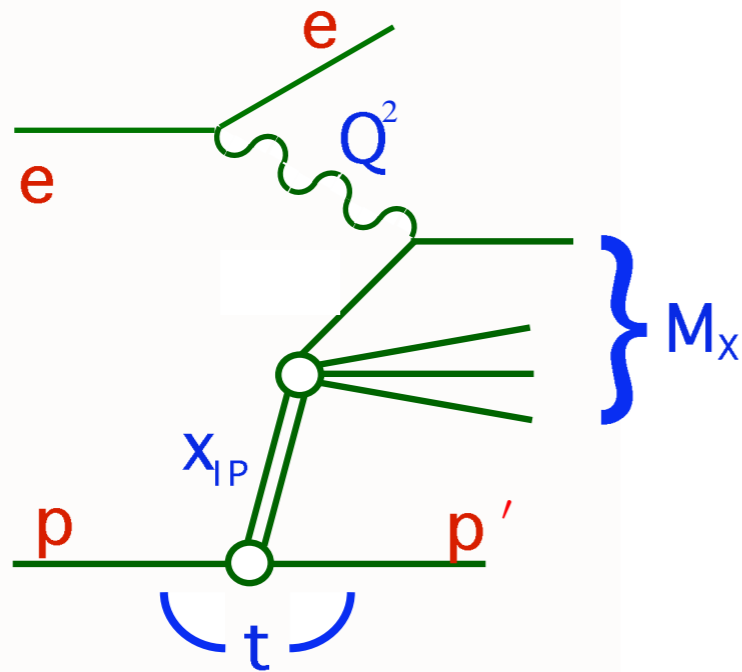




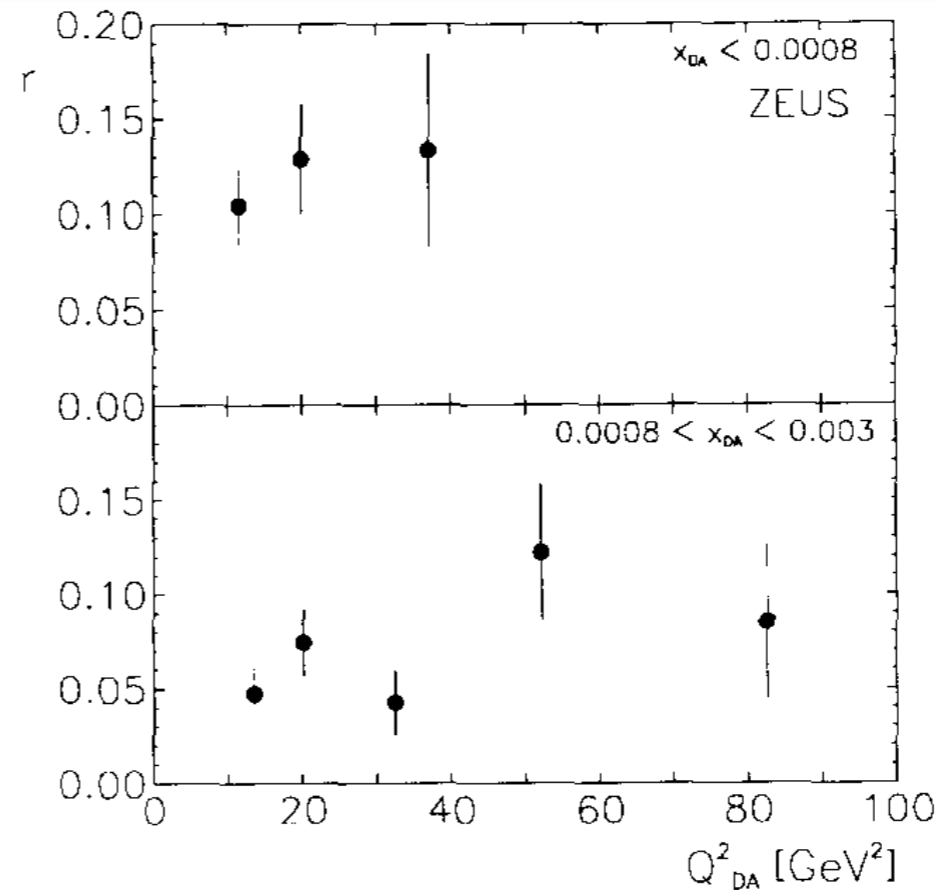
# Features of BLM/PMC

- **Predictions are scheme-independent at every order**
- **Matches conformal series**
- **No  $n!$  Renormalon growth of pQCD series**
- **New scale appears at each order;  $n_F$  determined at each order - matches virtuality of quark loops**
- **Multiple Physical Scales Incorporated (Hoang, Kuhn, Tuebner, sjb)**
- **Rigorous: Satisfies all Renormalization Group Principles**
- **Realistic Estimate of Higher-Order Terms**
- **Reduces to standard QED scale  $N_C \rightarrow 0$**
- **GUT: Must use the same scale setting procedure for QED, QCD**
- **Eliminates unnecessary theory error**
- **Maximal sensitivity to new physics**
- **Commensurate Scale Relations between observables: Generalized Crewther Relation (Kataev, Lu, Rathsmann, sjb)**
- **PMC Reduces to BLM at NLO: Example: BFKL intercept (Fadin, Kim, Lipatov, Pivovarov, sjb)**

# Remarkable observation at HERA



*10% to 15%  
of DIS  
events are  
diffractive!*

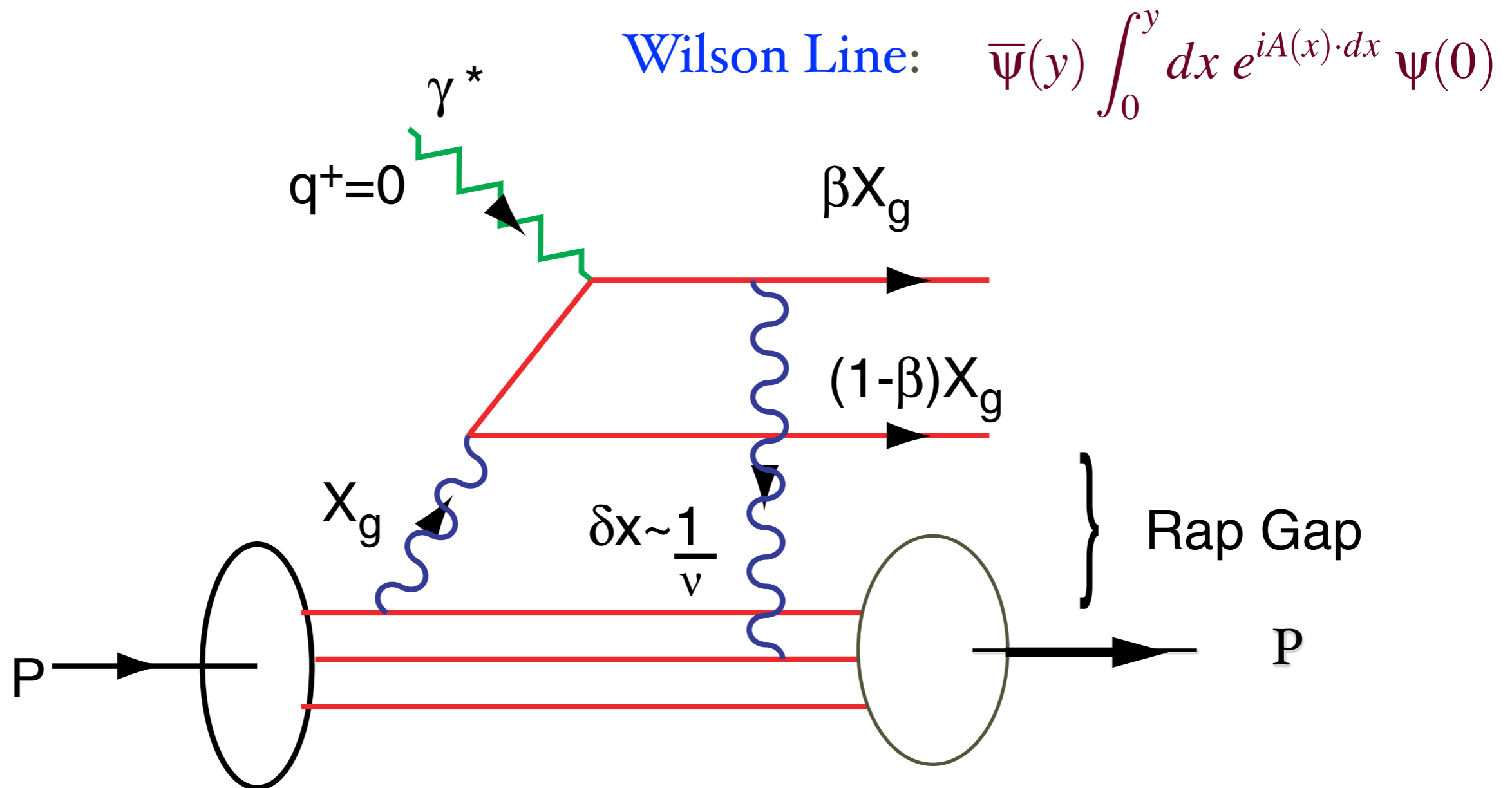


Fraction  $r$  of events with a large rapidity gap,  $\eta_{\max} < 1.5$ , as a function of  $Q_{DA}^2$  for two ranges of  $x_{DA}$ . No acceptance corrections have been applied.

M. Derrick et al. [ZEUS Collaboration], Phys. Lett. B 315, 481 (1993)

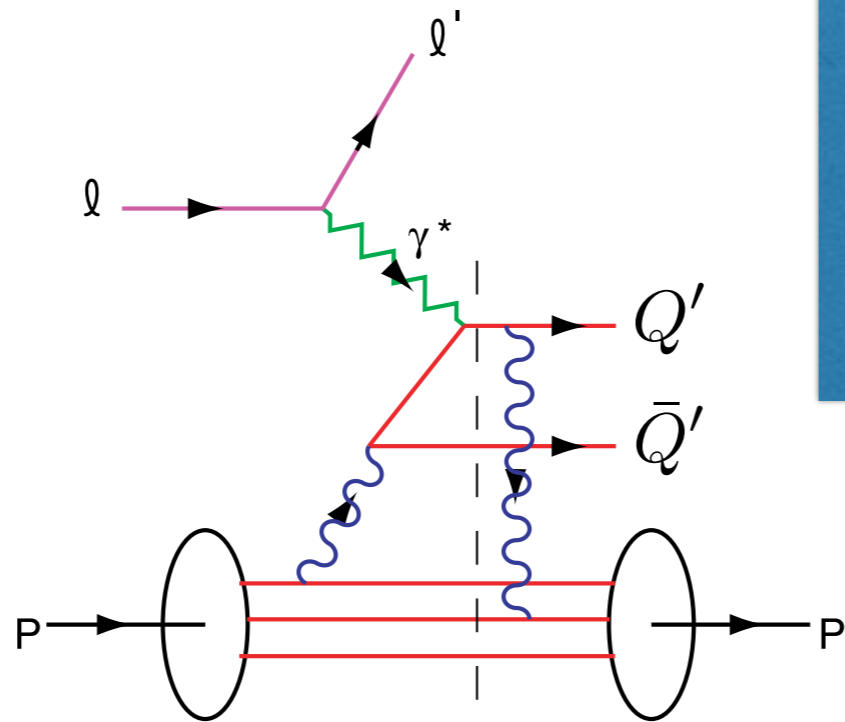


# QCD Mechanism for Rapidity Gaps



**Reproduces lab-frame color dipole approach**  
**DDIS: Input for leading twist nuclear shadowing**

DDIS:  
Diffractive  
Deep Inelastic  
Scattering



**90% of proton momentum carried off  
by final state  $p'$  in 15% of events!**

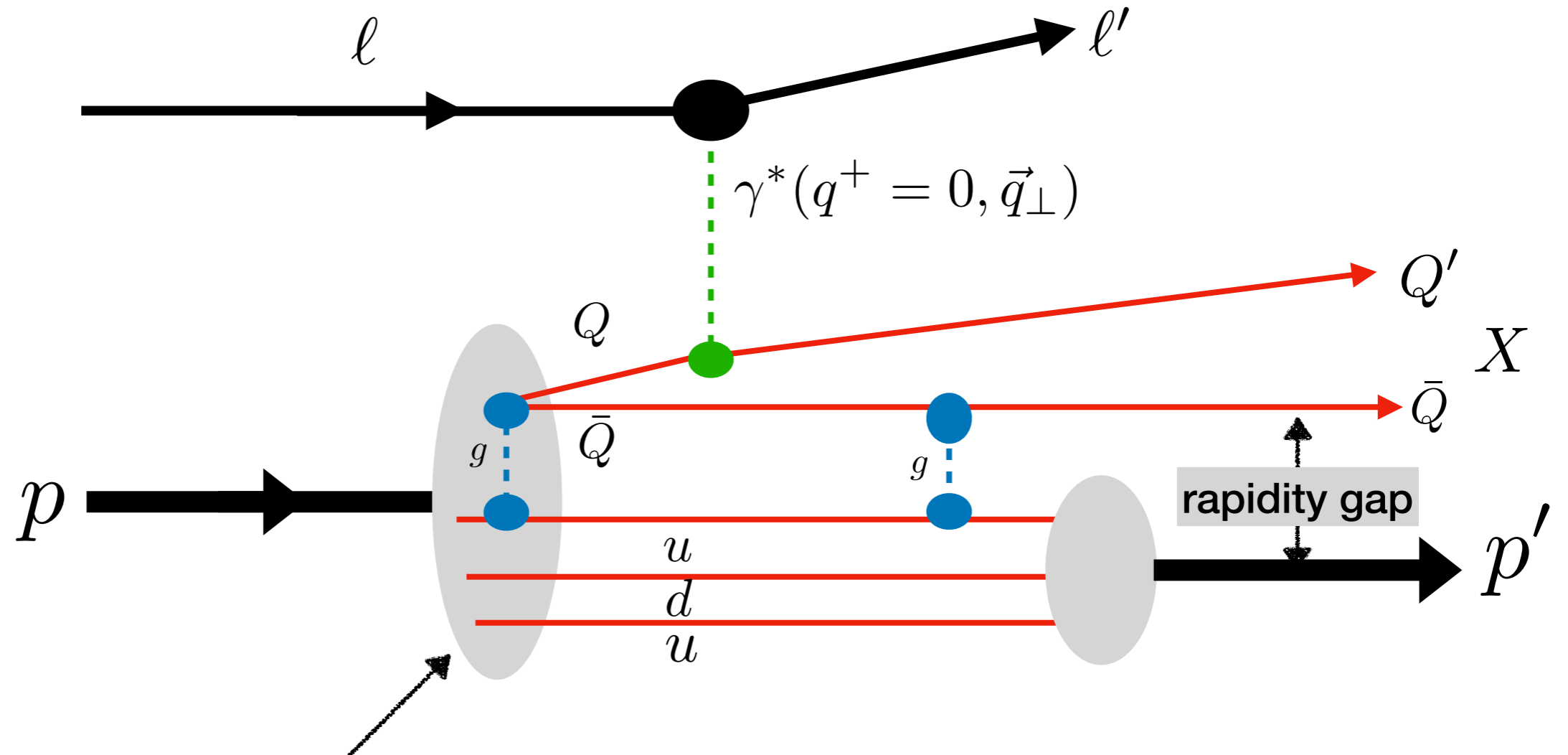
**Gluon momentum fraction misidentified!**

$p'$  is measured in DDIS but escapes detection in DIS events

**Simplified Description of DDIS from two-gluon Pomeron exchange in the LF framework**

**Five-quark Fock State + final-state interaction produces rapidity gap**

Diffractive DIS Event:  $\gamma^* + p_{|uduQ\bar{Q}\rangle} \rightarrow p' + X + (rapgap)$

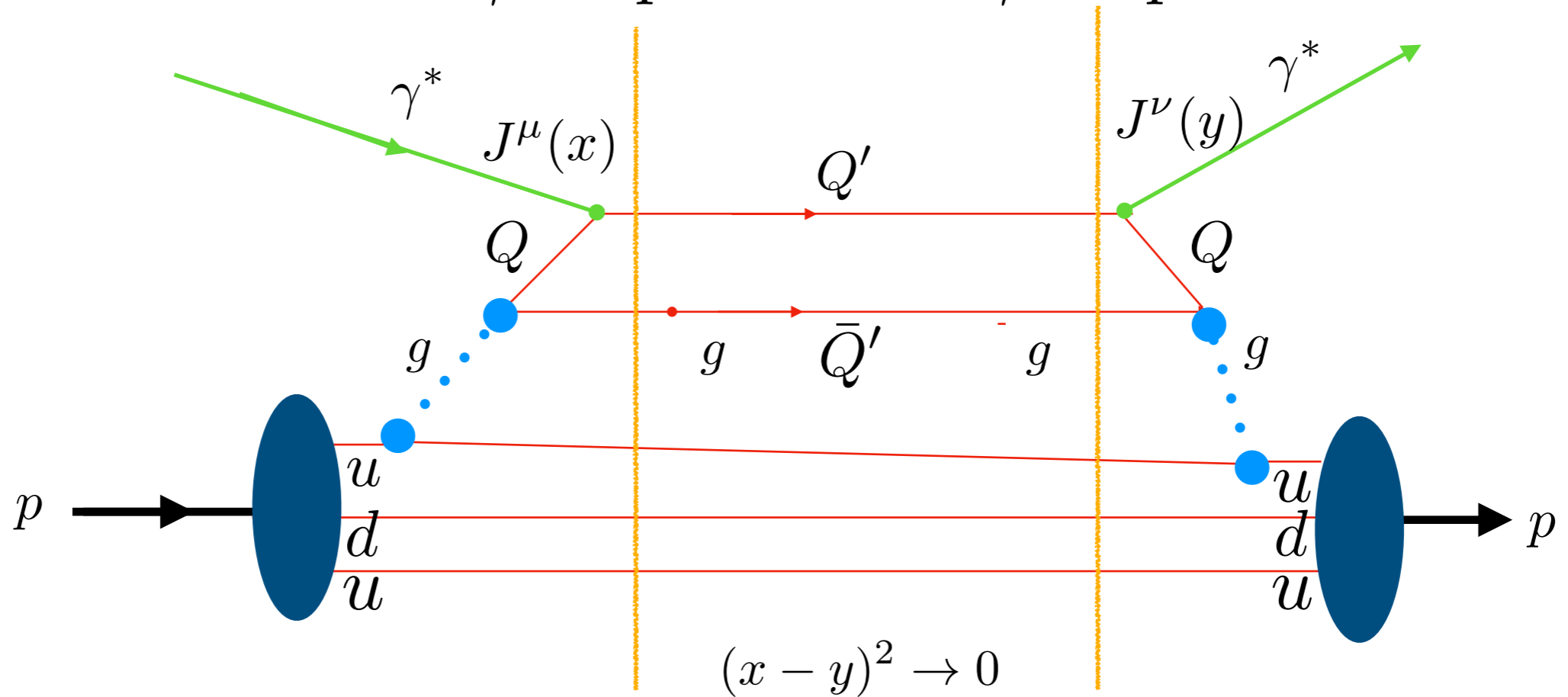


Five-quark Fock state of proton:  $|\{udu\}_{8C}\{Q\bar{Q}\}_{8C}\rangle$

**Low-Nussinov Two-Gluon Model of Pomeron**

*Forward Virtual Compton scattering for a DIS event*

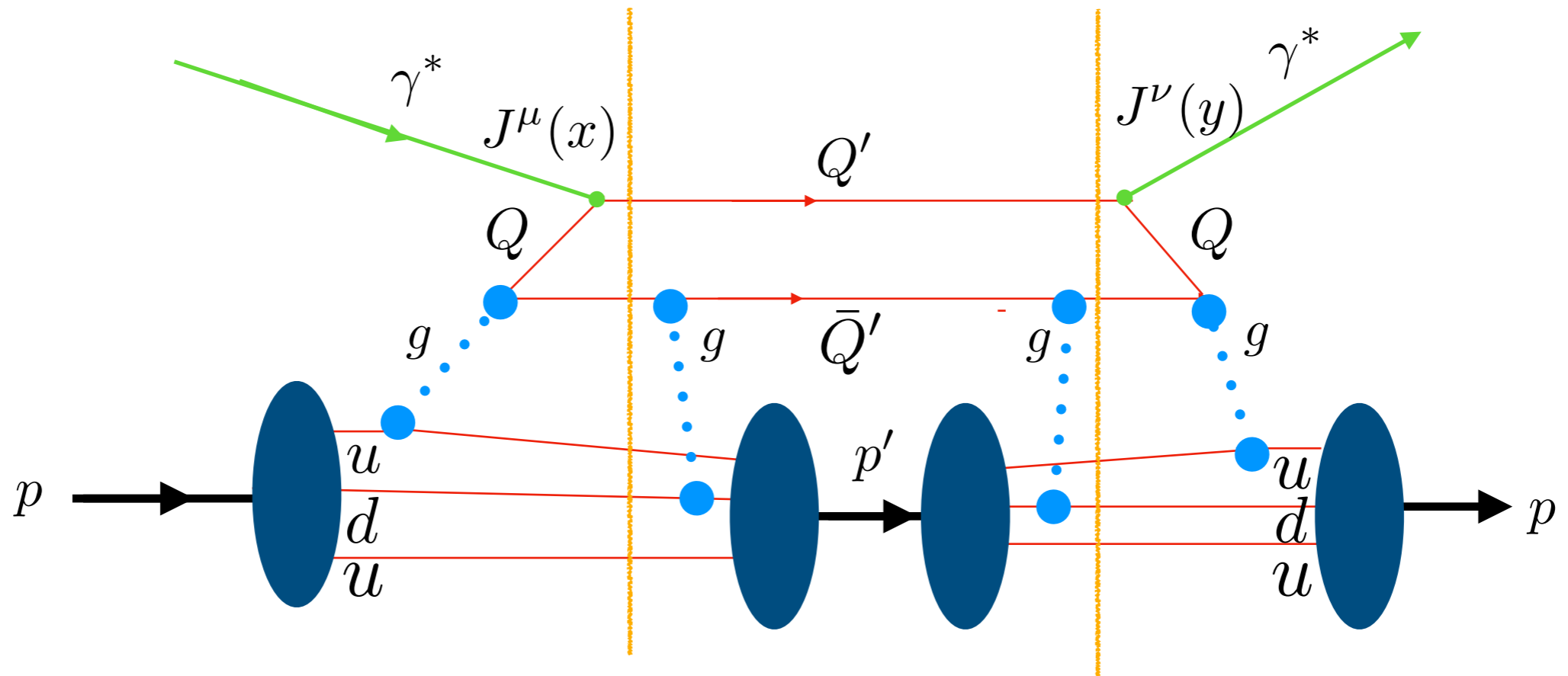
$$\gamma^* + p \rightarrow X \rightarrow \gamma^* + p$$



Vanishing LF time between currents of virtual photons at large  $q^2$  : OPE!

*Forward Virtual Compton scattering for a DDIS event*

$$\gamma^* + p \rightarrow \{Q' \bar{Q}'\} + p' \rightarrow \gamma^* + p$$



**Nonzero LF propagation time between virtual photons: No OPE!**

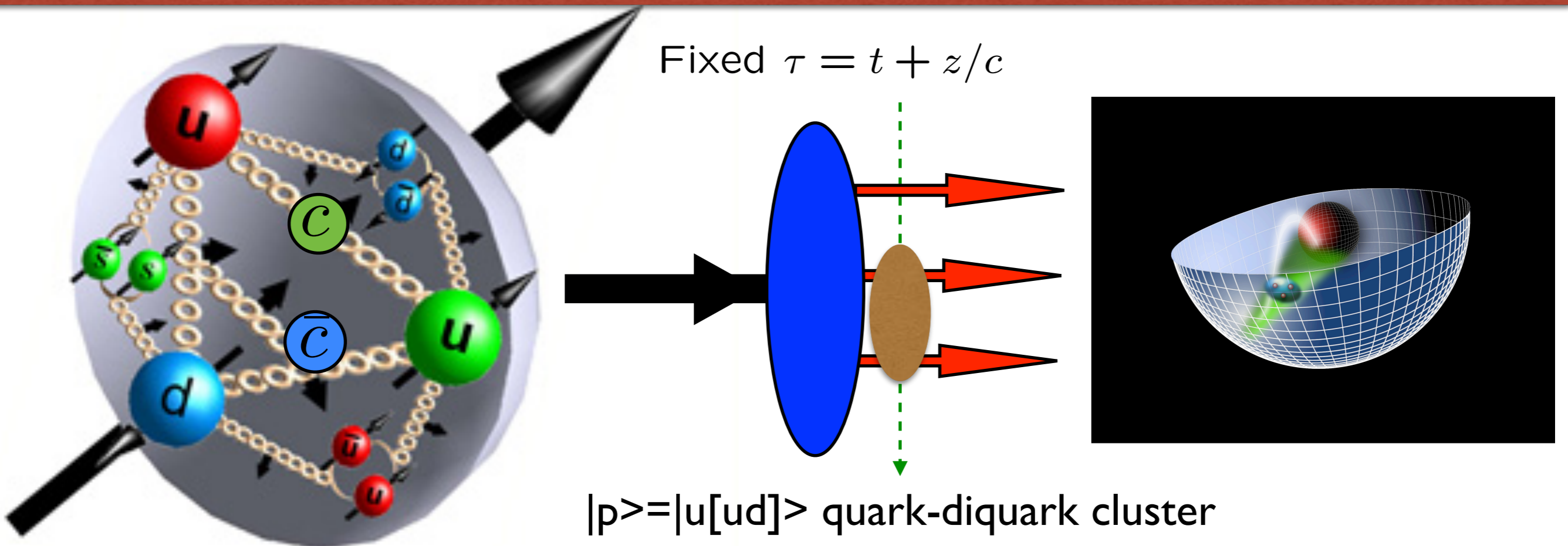
$$\langle p | J^\mu(x) | N \rangle \langle N | J^\nu(y) | p \rangle, (x - y)^2 \neq 0$$

**Cannot reduce to matrix element of local operator! No Sum Rules!**

Liuti, Lubovitski,  
Schmidt, sjb



# Color Confinement and Supersymmetric Features of Hadron Physics from Light-Front Holography and Novel Features of QCD from Light-Front Holography II



with Guy de Tèramond, Hans Günter Dosch, Alexandre Deur, Marina Nielsen, Ivan Schmidt, F. Navarra, Jennifer Rittenhouse West, G. Miller, Keh-Fei Liu, Tianbo Llu, Liping Zou, S. Groote, Joshua Erlich, S. Koshkarev, Xing-Gang Wu, Sheng-Quan Wang, Cedric Lorcè, R. S. Sufian, R. Vogt, G. Lykasov, S. Gardner, S. Liuti, V. Lyubovitskij, L. di Giustino

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Beyond the Standard  
Models?

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**Talk II July 8, 2021**